Plasma evolution towards critical equilibria and diamagnetism

by Emilia R. Solano
Laboratorio Nacional de Fusión,
CIEMAT, Madrid, Spain

Equilibrium (Grad-Shafranov) bifurcation

$$G(\Psi) \equiv \frac{1}{\mu_0 R}\left(\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial \Psi}{\partial R} + \frac{\partial^2 \Psi}{\partial Z^2}\right) + \left(R\,p' + \frac{(F^2)'}{2\mu_0 R}\right) = 0$$

$$\equiv \qquad L(\Psi) \qquad + \qquad J(\Psi) \qquad = 0$$

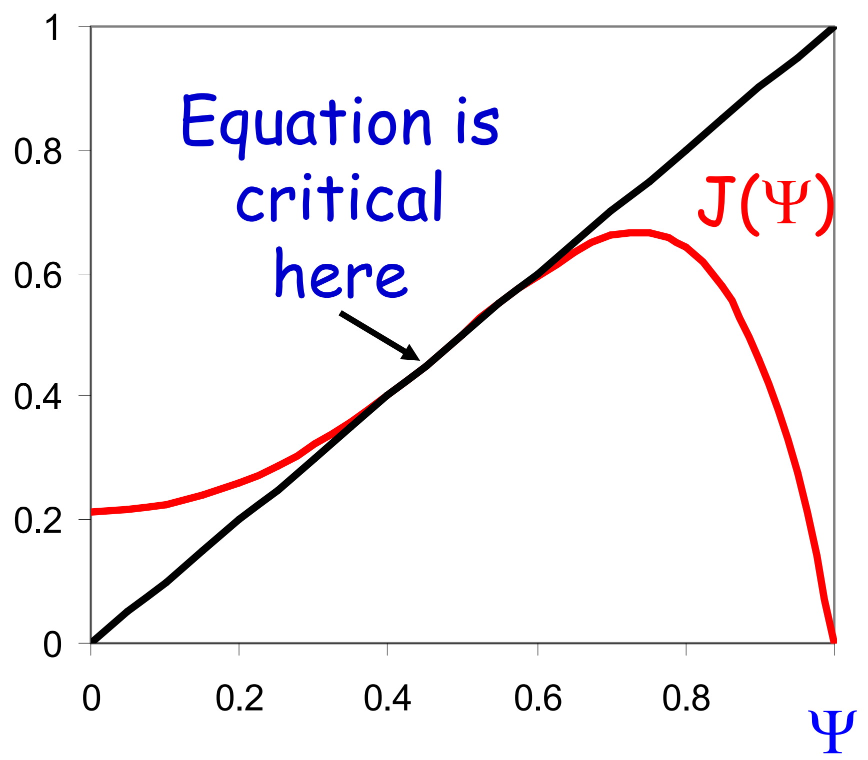
Mathematically, number of solutions of equation can change when:

$$\boxed{-J(\Psi) + \Psi J_\Psi(\Psi) = 0}$$

For instance, when

$p'(\Psi_c) = a+b(\Psi-\Psi_c)$

$FF'(\Psi_c) = c(\Psi-\Psi_c)^n$, $n \geq 2$



Diamagnetism

$p'(\Psi_c) \;\; = a + b(\Psi - \Psi_c)$

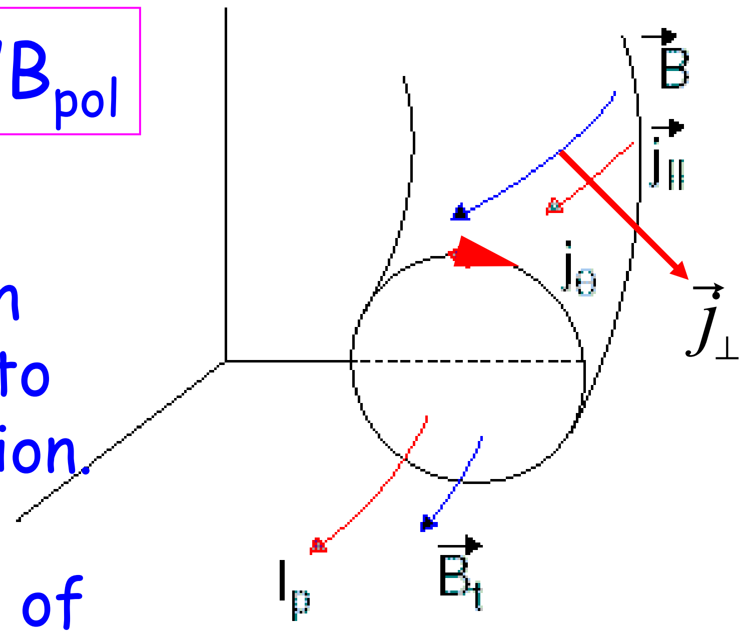
$FF'(\Psi_c) = c(\Psi - \Psi_c)^n, \; n \geq 2$

$\boxed{j_{pol} = -F' B_{pol}}$

A small perturbation of GS equation near a critical zero of FF' can lead to the formation of a diamagnetic region.

In a diamagnetic region, steepening of $\nabla\Psi$ (and ∇p) is compatible with force balance:

$$\boxed{j_{tor} \times B_{pol} + j_{pol} \times B_{tor} = \nabla p}$$



At critical point j is toroidal

Plasma evolution?

the usual transport way to evolve the plasma

- plasma is always assumed to be in equilibrium, $p=p(\Psi)$
- τ_E : $n(\Psi,t)$, $T(\Psi,t)$, $p(\Psi,t)$ diffuse in Ψ space,
- diffusion coefficients given by collisions & μturbulence.
- $\tau_{L/R}$: equilibrium $\Psi(R,Z,t)$ evolves with fixed $p(\Psi)$, $F(\Psi)$

Another way?

- plasma is always assumed to be in equilibrium, $p=p(\Psi)$
- Assume $n(\Psi)$, $T(\Psi)$, $p(\Psi)$ stationary in Ψ space
- Add localised parallel current to the plasma
- Watch evolution of currents and Ψ
- Profiles ride on Ψ: Ψ barriers look like transport barriers

Resistive magnetic plasma evolution can be fast

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}),$$

ignore vxB term

In a cylinder, j_{pol}, j_{tor} obey heat diffusion-like equations

non-inductive current drive

$$\frac{\partial j_{tor}}{\partial t} - \left(\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \right) \eta (j_{tor} - j_{tor,ni}) = 0$$

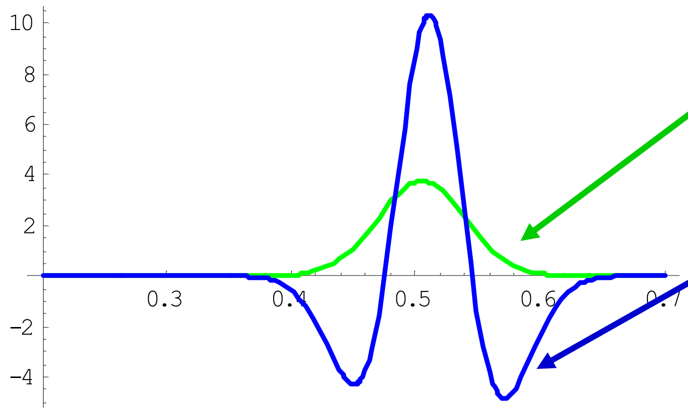
$$\frac{\partial j_{pol}}{\partial t} - \left(\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{1}{r^2} \right) \eta (j_{pol} - j_{pol,ni}) = 0$$

Study current evolution with non-inductive CD (bootstrap, or externally applied CD source).

Magnetic plasma evolution

Most current drive in a tokamak drives co-parallel current.

But there is a fast transient effect at switch-on



Gaussian current source

transient back emf

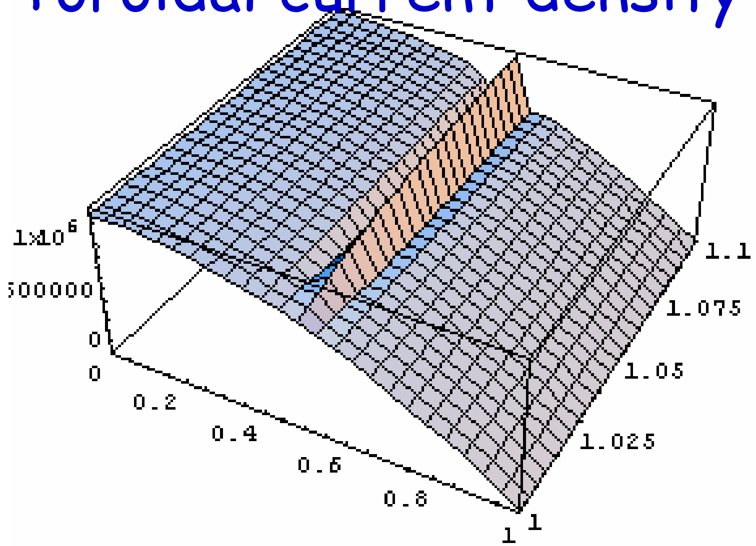
$$\left.\frac{\partial j}{\partial t}\right|_{\text{Source Term}} = -\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \frac{1}{r^2}\right)\frac{\eta(r)}{\mu_0}e^{\frac{-2(r-r_0)^2}{w_o}}$$

Current diffusion: towards criticality condition and diamagnetism in plasma

Simplest model: cylindrical plasma,
model Te(r), resistivity,
add localised parallel current source
(Gaussian, r_0=.5,w_0=.1)

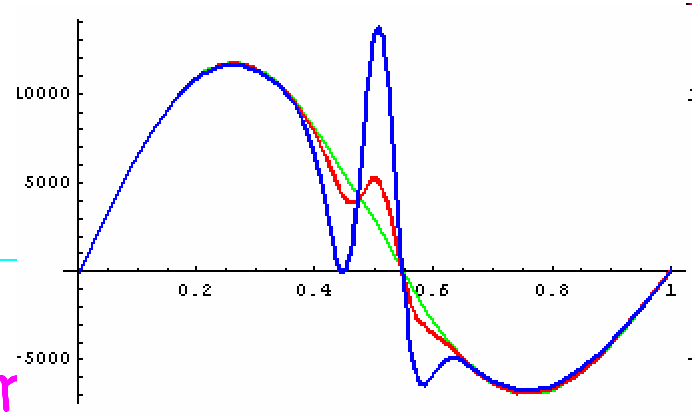
j_{niCD}=.3 MA/m^2

Toroidal current density



Toroidal current evolution:
j_{tor} init parabolic
j_{tor} profile -> criticality in 10 ms

Poloidal current density



Poloidal current evolution:
j_{pol} init semi-paramagnetic,
j_{pol} profile -> local diamagnetism in 40 ms

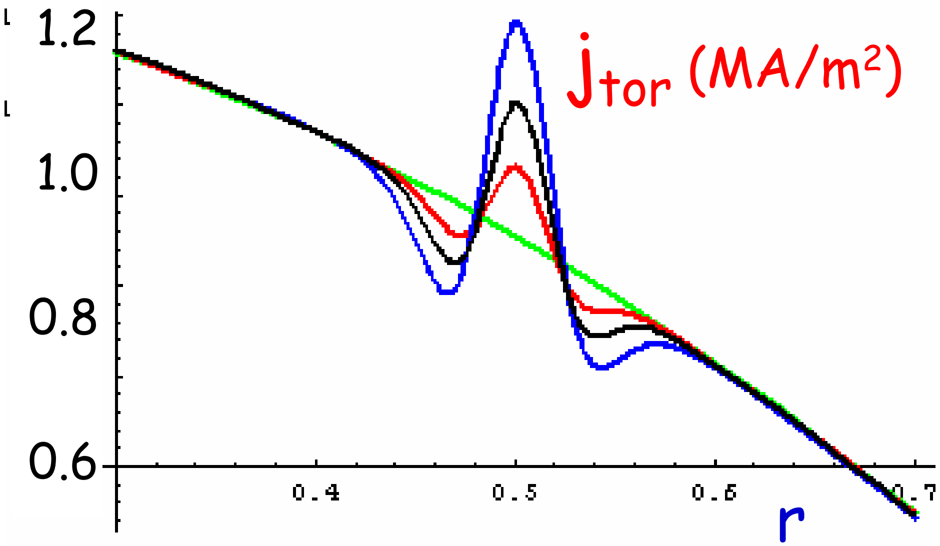
The system evolves naturally towards criticality and diamagnetism

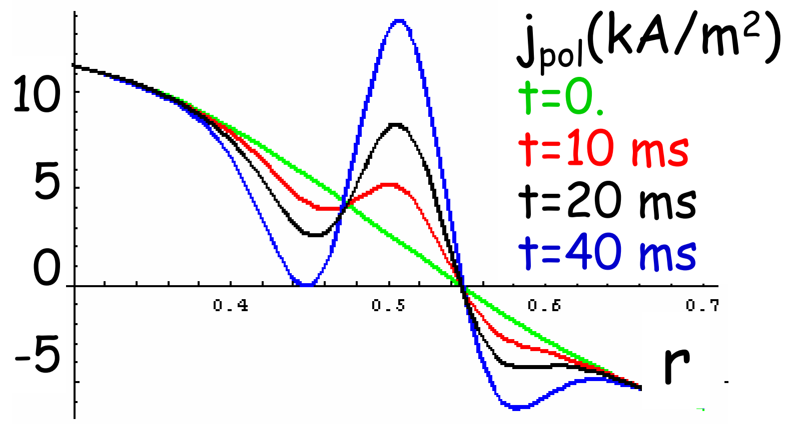
Current diffusion: CD-induced ITB?

Response time to local non-inductive current drive is fast:

criticality: 10 ms, Ψ=.2, r=.45

diamagnetism at r=.45: 40 ms





The system evolves naturally towards criticality and diamagnetism

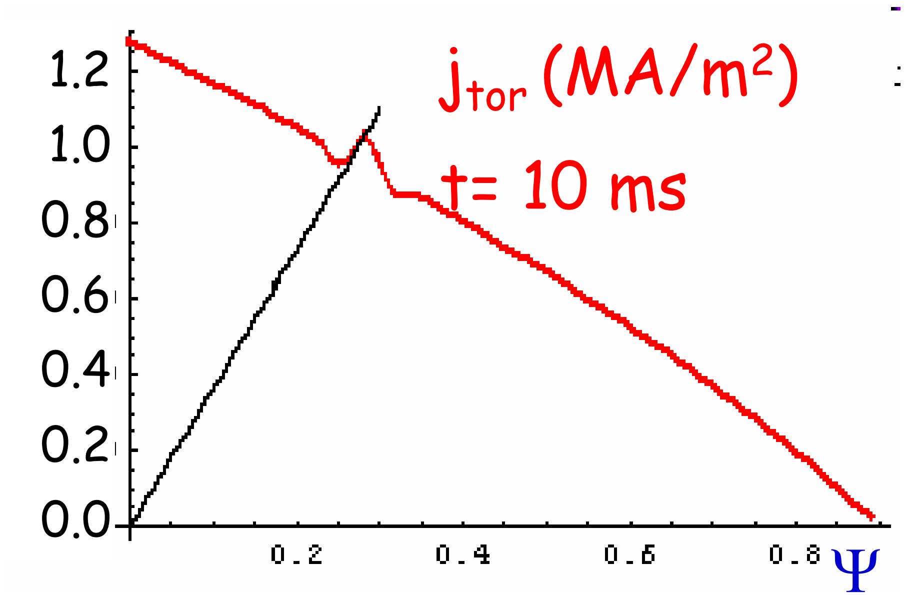
but in this case shear reverses 1st...

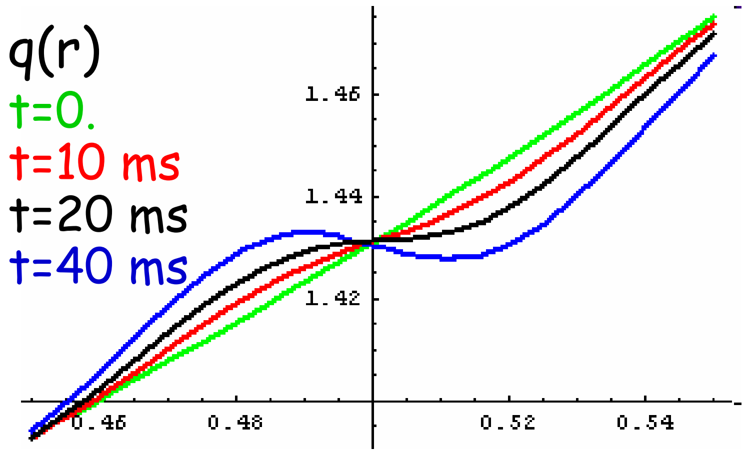
Order of events?

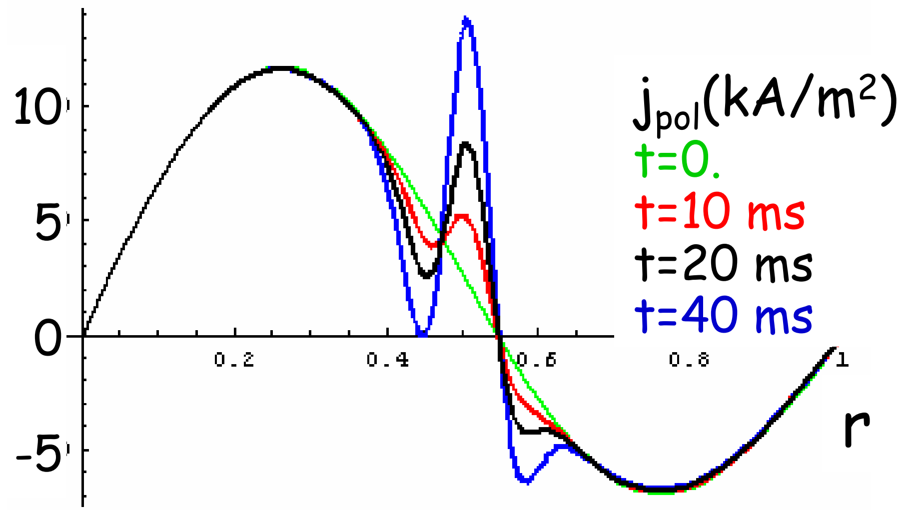
criticality at Ψ=.3, r=.45, 10 ms

shear reversal at r=0.5, 20 ms

diamagnetism at r=.45, 40 ms







Relative order of events depends on initial conditions ...

Where is the bifurcation?

Not so fast... the work has just begun.

1. Constant Te(r) does not really satisfy constant Te(Psi), so energy equation is uncontrolled.
2. In these simulations Ip(t) was uncontrolled, so confinement itself is changing
3. Experimentally difficult to distinguish various conditions.

We need to solve the **coupled** system of equations, Te(Psi,t) and poloidal and toroidal current evolution, including feedback current control, to see if bifurcation will occur at some point.

Possible models for physics of barriers:

Resistive plasma evolution drives plasma towards local criticality, diamagnetism and shear reversal: TBs may be hard to avoid.

The available experimental evidence is compatible with "Psi barriers" and "diamagnetic barriers":

- often all profiles become more peaked at the same radial location, simultaneously
- after a diamagnetic solution branch develops, turbulence may be reduced by Shafranov-shift stabilisation or by steeper electric fields: transport is reduced, "transport" barriers can form.

ECCD or LHCD for local ITB control?

Back to Maths:

The criticality condition I derived for semilinear PDEs may well have wider applicability in mathematical physics.

I would be very interested if anyone knows a solved model non-linear problem in which we can test the simple criticality condition.

Note: if N is a polynomial without a constant term, $N_u=0$ is trivially verified by $u=0$, rendering the criticality condition fairly useless.

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