Magnetic phase transition model for L to H transition

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Plasma magnetization in a tokamak

Plasma force balance: $\nabla p = \vec{j} \times \vec{B} = \vec{j}_\zeta \times \vec{B}_\theta + \vec{j}_\theta \times \vec{B}_\zeta$

In cylindrical approximation : $\frac{d}{dr}\left(p + \frac{B_z^2 + B_\theta^2}{2\mu_0}\right) = -\frac{B_\theta^2}{r\mu_0}$

Integrating:

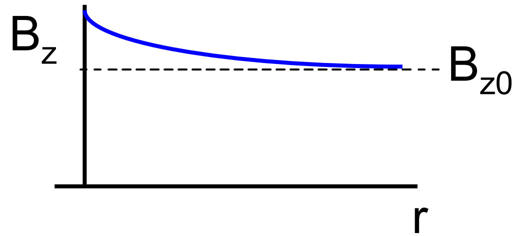
$$\beta_\theta \equiv \frac{\int_0^a p dS}{B_{\theta a}^2 / 2\mu_0} = \frac{B_{za}^2 - \langle B_z^2 \rangle}{B_{\theta a}^2} \simeq 1 + \frac{2B_{za}\left(B_{za} - \langle B_z \rangle\right)}{B_{\theta a}^2}$$

β_θ is related to average plasma magnetisation:

$\beta_\theta < 1$

$\nabla p < \vec{j}_\zeta \times \vec{B}_\theta$

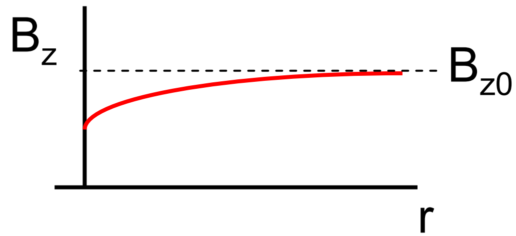
B_z increased by j_θ
paramagnetism,
low pressure

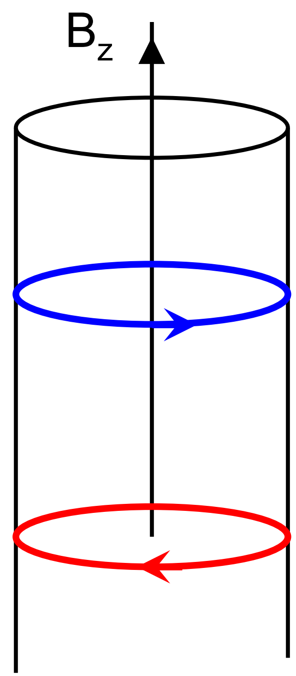


$\beta_\theta > 1$

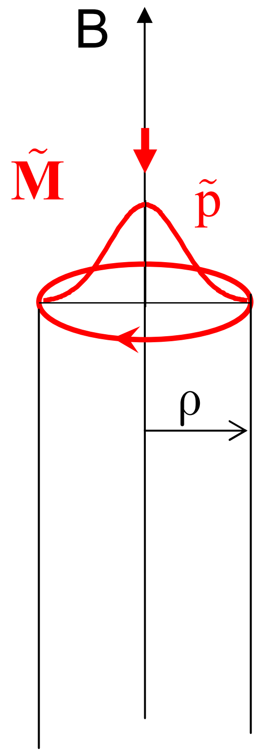
$\nabla p > \vec{j}_\zeta \times \vec{B}_\theta$

B_z reduced by j_θ
diamagnetism,
high pressure





Magnetism in cylindrical blob with pressure hill/hole

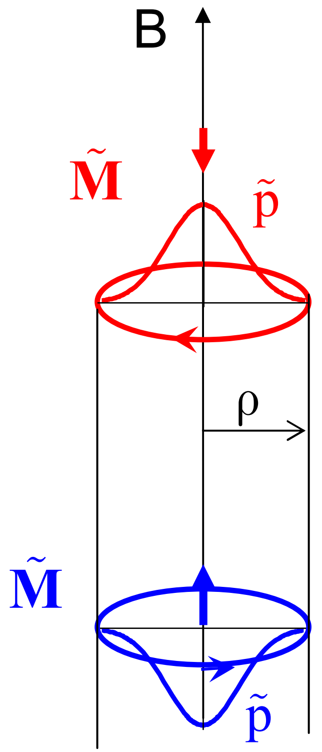


$$\mathbf{F} = mn\frac{d\mathbf{v}}{dt} = -\nabla\tilde{p} + \mathbf{j}\times\mathbf{B} \qquad \tilde{\mathbf{j}}_\perp = \frac{\mathbf{b}\times\nabla\tilde{p}}{B}$$

Diamagnetic current:

if inside the tube there is a pressure hill, the associated perpendicular current reduces B_z: diamagnetism

Magnetism in cylindrical blob with pressure hill/hole



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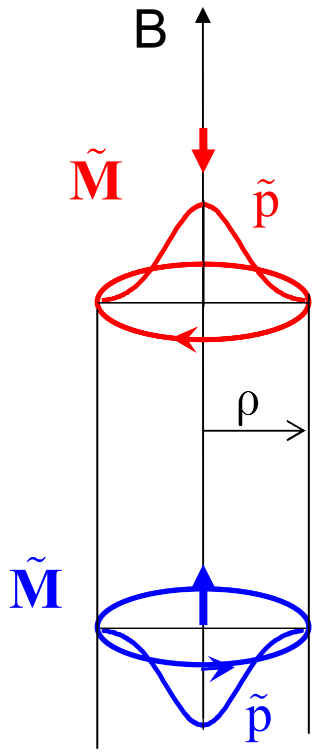
Diamagnetic current:
if inside the tube there is a pressure hill, the associated perpendicular current reduces B_z: diamagnetism

Paramagnetic current:
if inside the tube there is a pressure hole, the associated perpendicular current increases B_z: paramagnetism

Magnetism in cylindrical blob with pressure hill/hole



$$\mathbf{F} = mn\frac{d\mathbf{v}}{dt} = -\nabla\tilde{p} + \tilde{\mathbf{j}}\times\mathbf{B} = 0 \qquad \tilde{\mathbf{j}}_\perp = \frac{\mathbf{b}\times\nabla\tilde{p}}{B}$$

Diamagnetic current:
if inside the tube there is a pressure hill, the associated perpendicular current reduces B_z: diamagnetism

Paramagnetic current:
if inside the tube there is a pressure hole, the associated perpendicular current increase B_z: paramagnetism

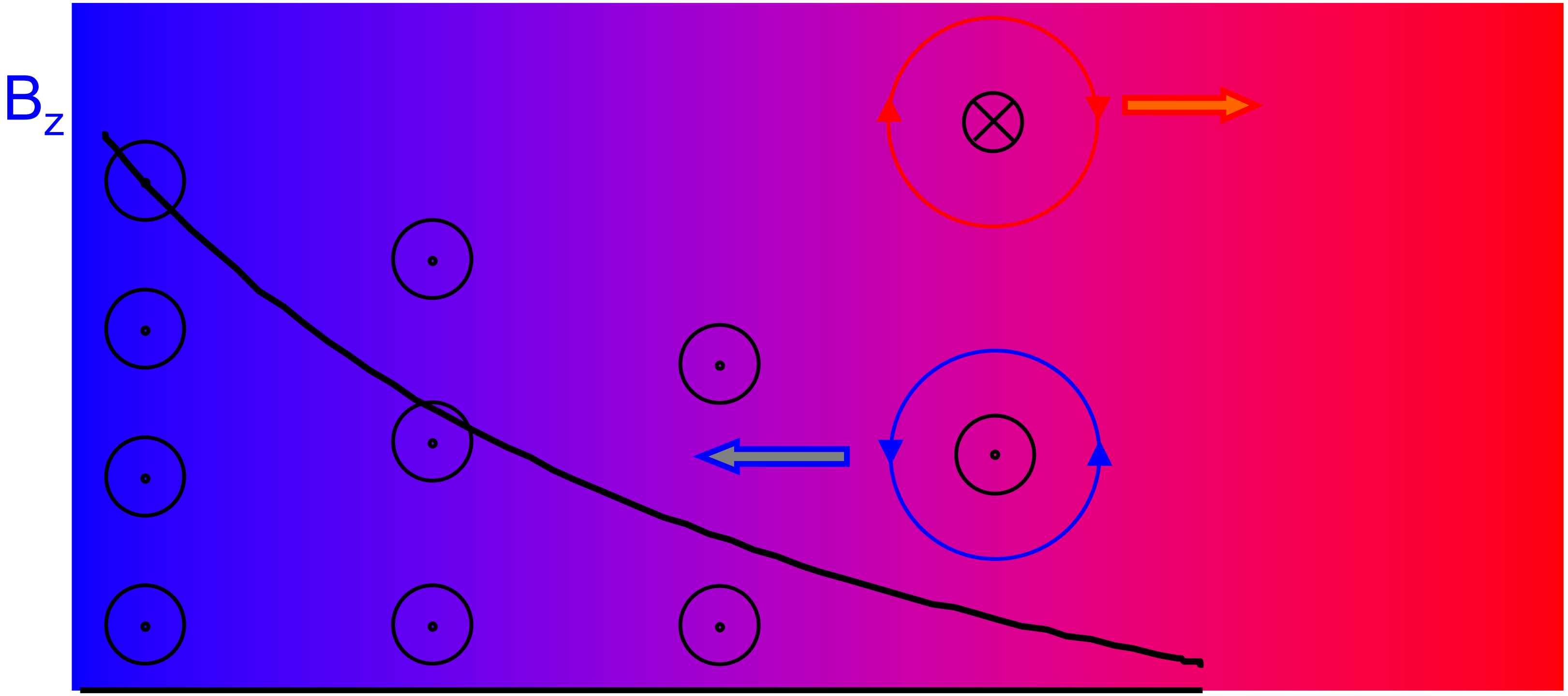
Magnetization of the blob:

$$\nabla\times\mathbf{M} = \mu_0\frac{b\times\nabla\tilde{p}}{B} = -\frac{dM}{dr}\hat{\mathbf{r}}$$

$$\tilde{\mathbf{M}} = \frac{1}{\lambda_\parallel}\int_0^\rho \frac{\mathbf{b}}{B}\frac{\partial\tilde{p}(\rho')}{\partial\rho'}\lambda_\parallel\, d\rho' \approx -\frac{\tilde{p}}{\overline{B}}\mathbf{b} \quad \begin{cases} < 0, \text{ dia} \\ > 0, \text{ para} \end{cases}$$

Movement of magnetized object in field gradient

$$F = \tilde{j} \times B$$



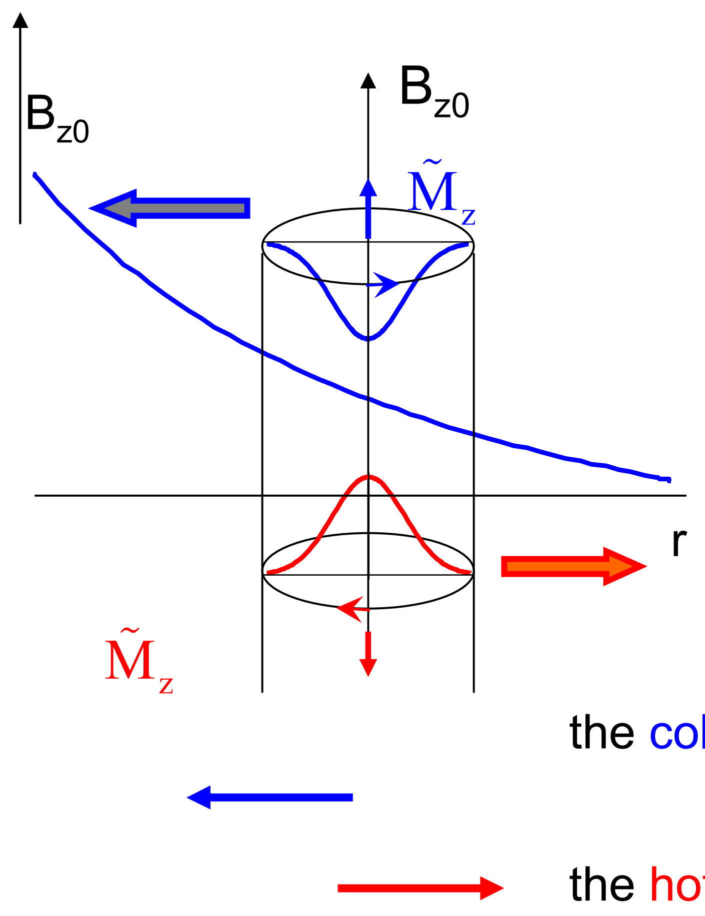
Field gradient created by paramagnetic current in background plasma:

Paramagnetic blobs attracted to high field region.

Diamagnetic blobs attracted to low field region.

Movement of magnetized object in field gradient

(see Jackson)



$$m_V \frac{d\mathbf{v}}{dt}\bigg|_V = \int \left(\nabla(\tilde{\mathbf{M}}.\mathbf{B})\right) dV \simeq$$

$$\boxed{\vec{B} = \vec{B}_0 + \vec{r}.\nabla\vec{B}_0 + \ldots}$$

$$\simeq \underbrace{\left(\int \left(\mathbf{r} \times \mathbf{j}_{mag}\right) dV\right)}_{\text{blob magnetization}} \int \nabla B_{0z} dV$$

$$m n_V \frac{d\mathbf{v}_r}{dt} \simeq \tilde{M}_z \nabla \overline{B}_{z0}$$

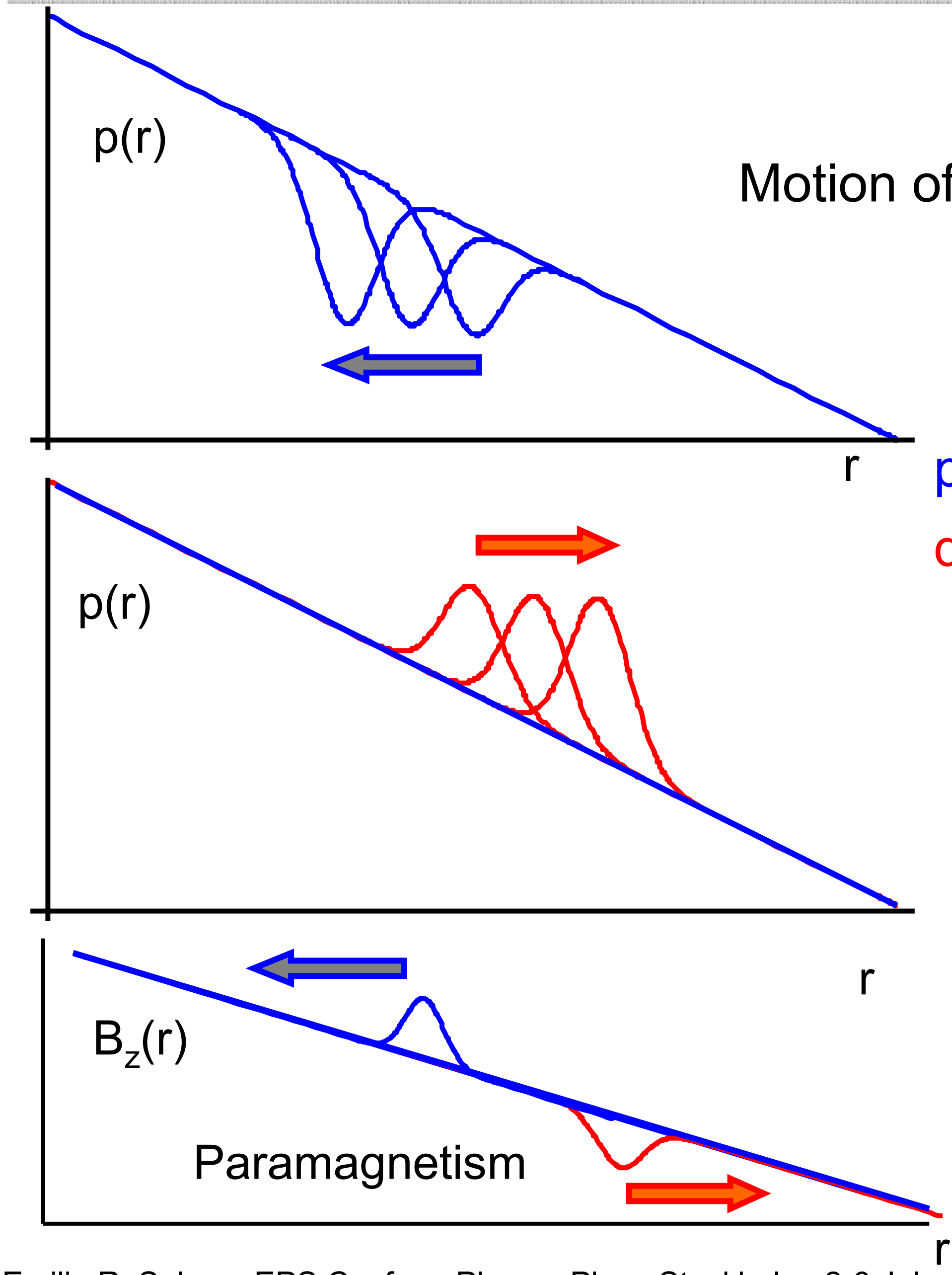
the cold blob (paramagnetic) seeks high field

the hot blob tube (diamagnetic) seeks low field

Blob averaged dB_z/dr controls motion of magnetised plasma blobs:

Anti-potential leads to ***magnetic phase separation***

Paramagnetic plasma: L-mode



Motion of pressure blobs depends on dB_z/dr

$$mn_V \frac{d\vec{v}_r}{dt} \simeq \tilde{M}_\zeta \nabla_r \bar{B}_{\zeta 0}$$

paramagnetic cold blobs move inward,

diamagnetic hot blobs move outward

outward thermal energy convection

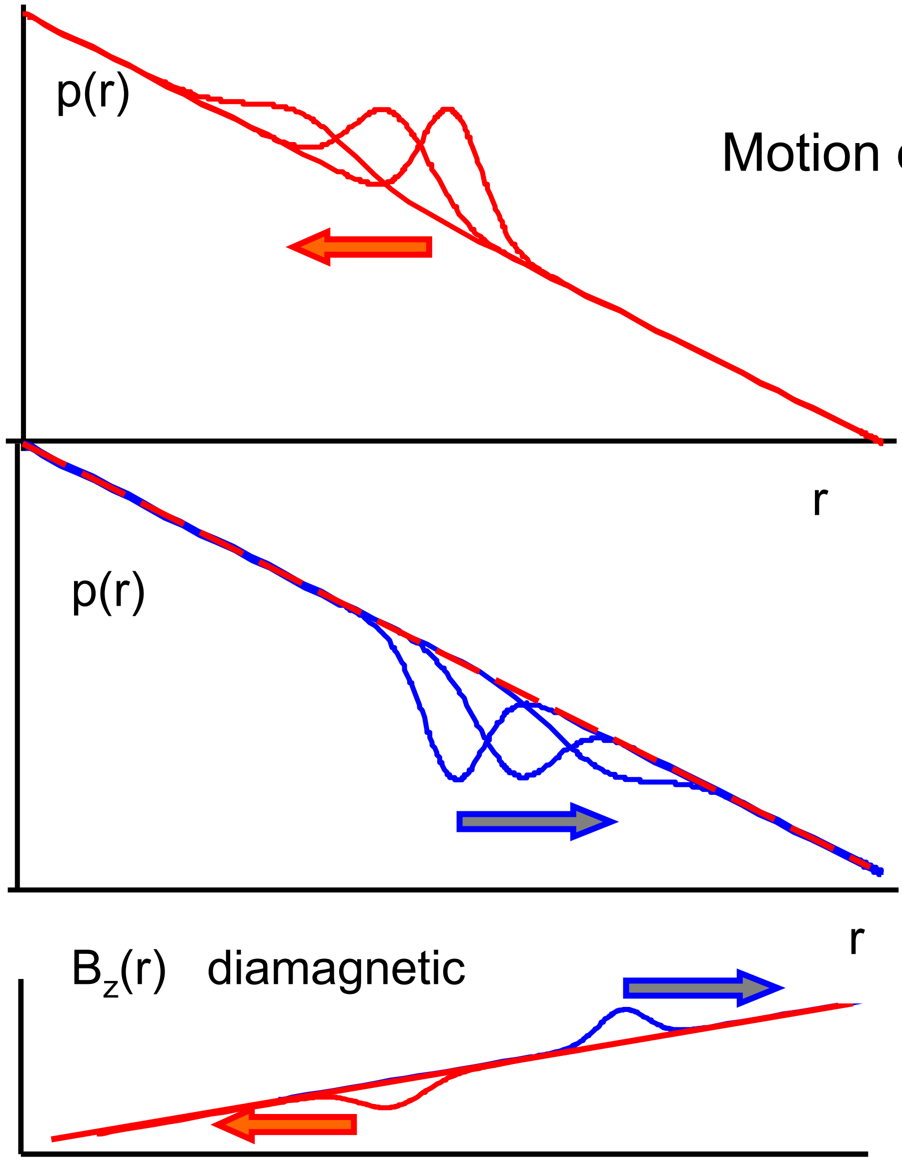
at the expense of

inward magnetic energy convection

p blobs “grow”, “instability”

L-mode

Diamagnetic plasma: H-mode



Motion of pressure blobs depends on dB_z/dr

$$mn_V \frac{d\vec{v}_r}{dt} \simeq \tilde{M}_\zeta \nabla_r \bar{B}_{\zeta 0}$$

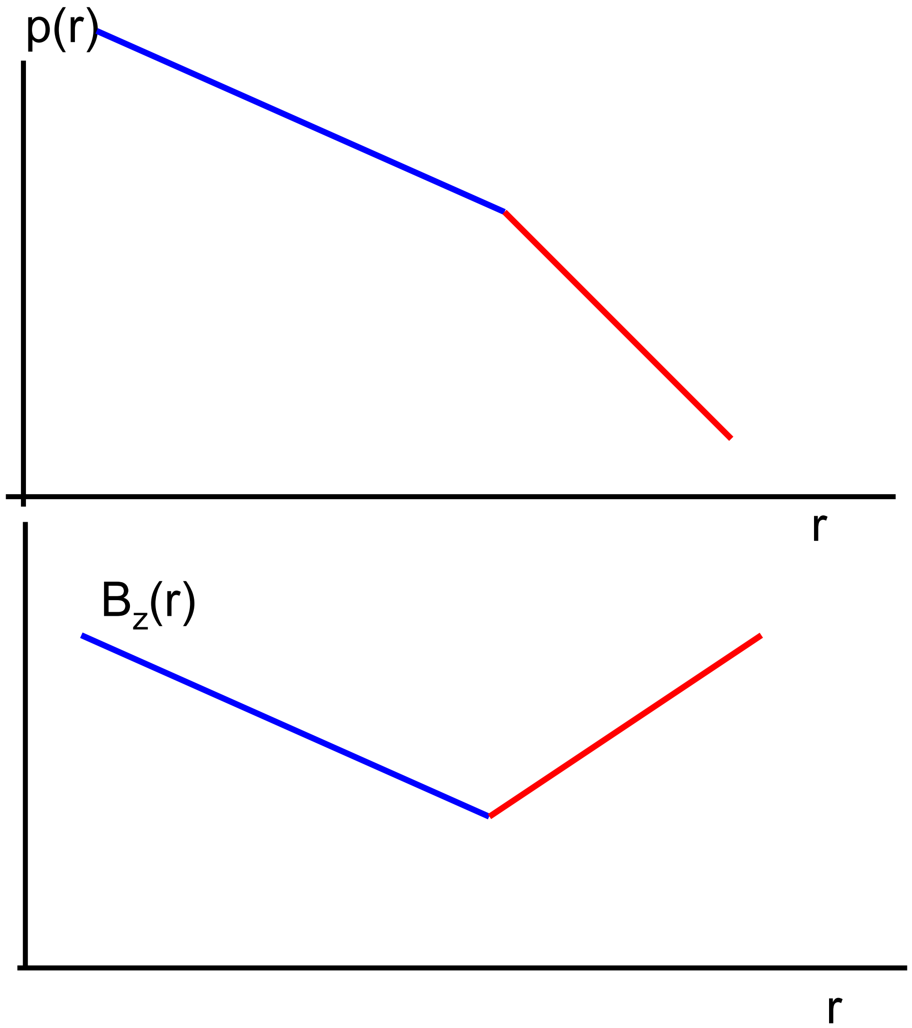
diamagnetic hot blobs move inward,
paramagnetic cold blobs move outward

inward thermal energy convection
at the expense of
outward magnetic energy convection

p blobs “decrease”, “saturation”

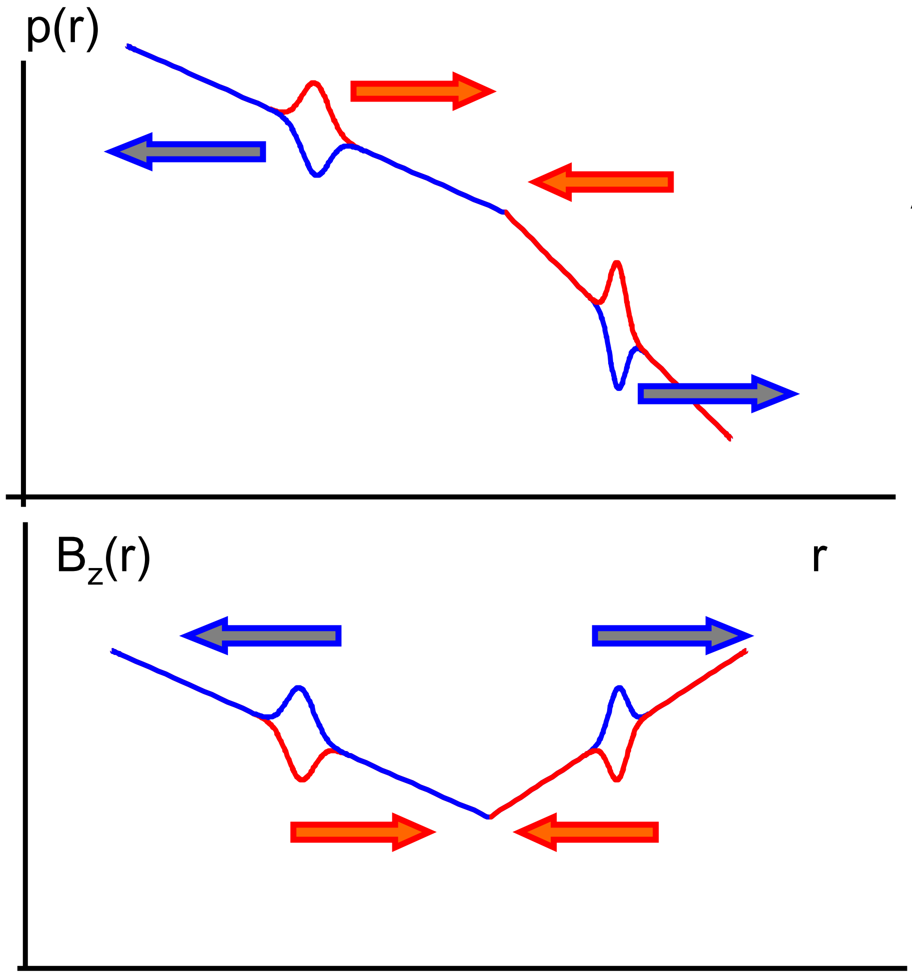
H-mode

Magnetic Boundary: phase transition



∇p increases somewhere, creating diamagnetic region at plasma edge.

Magnetic Boundary: phase transition



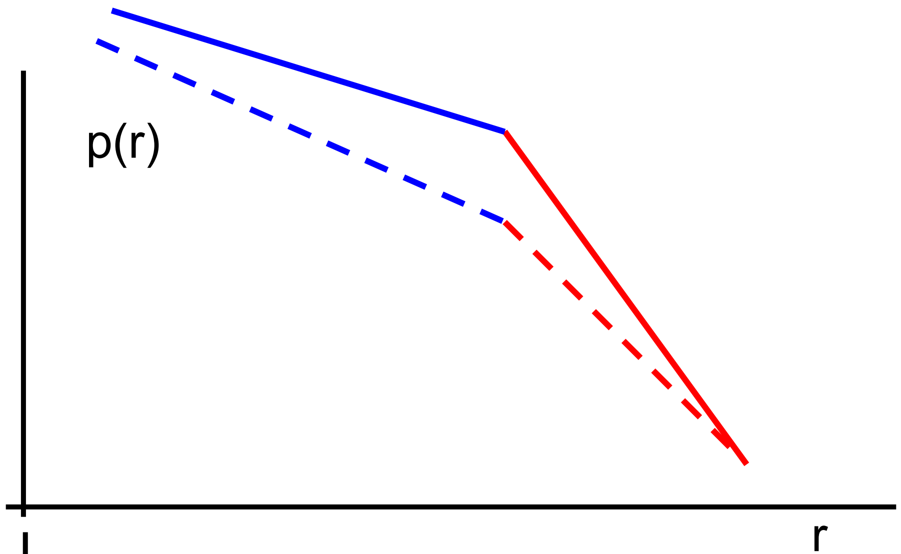
At a magnetic phase boundary blobs of the same type accumulate

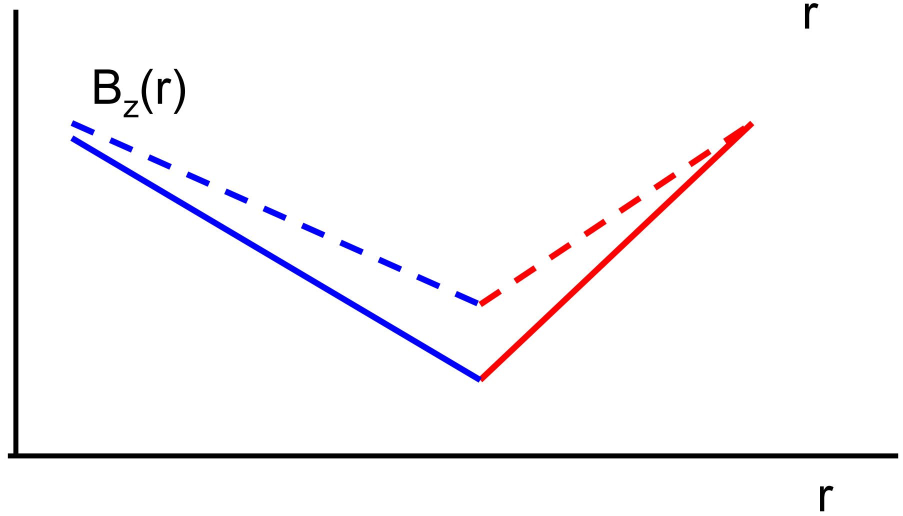
diamagnetic blobs (heat) seek wells

paramagnetic blobs seek hills

With multiple blobs moving, p and B_z profiles evolve

Magnetic Boundary: phase transition





∇p increases somewhere,
creating diamagnetic region
at plasma edge.

Magnetization,
of both signs, increases.

Phase transition is self-reinforcing.

Pressure pedestal forms, grows.

Pedestal formation at magnetisation boundary

Assume dashed $B_z(r)$, $p(r)$ initial profiles

Ideal MHD with <u>magnetization force</u>

$$\bar{n}_V m_i \frac{d^2\xi_r}{dt^2}\Big|_M = \tilde{M}_\zeta \nabla \bar{B}_{0z}$$

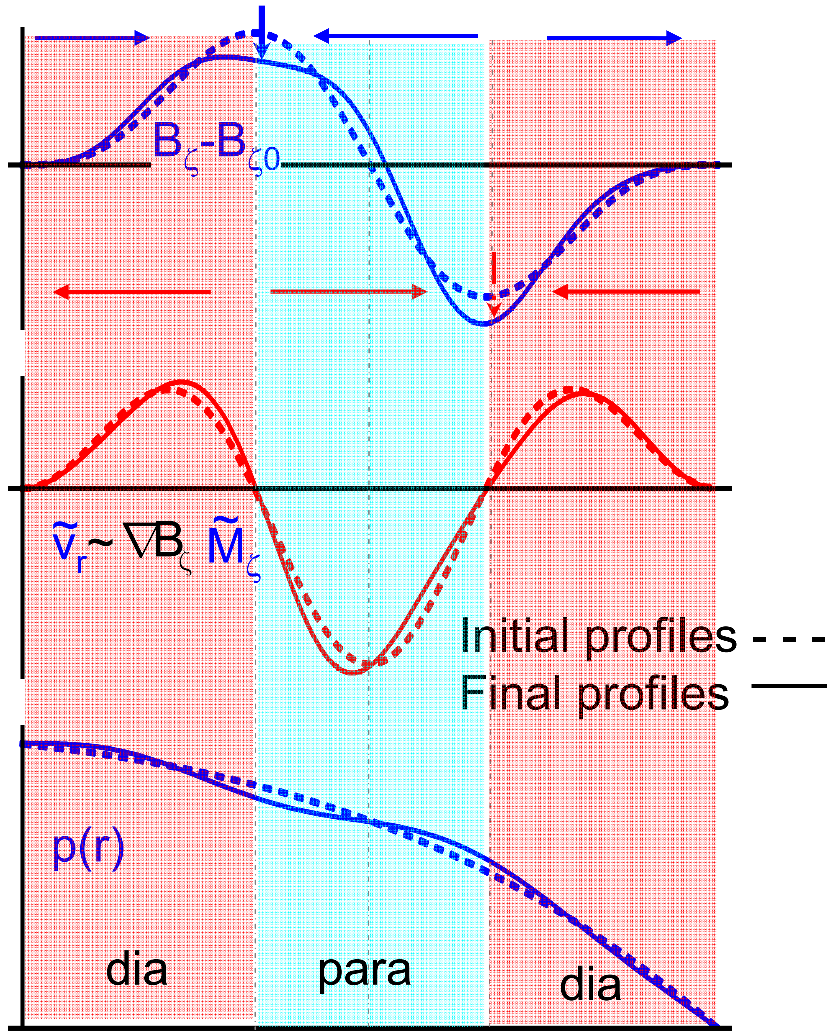
$$\frac{\partial B_z}{\partial t}\Big|_M = \nabla \times (\tilde{v}_r \bar{B}_{0z})$$

$$\frac{3}{2}\frac{\partial p}{\partial t}\Big|_M = -\nabla(\tilde{p}\,\vec{v})$$

Integrating one temporal step Δt

pressure steepens in diamagnetic regions, increases diamagnetism

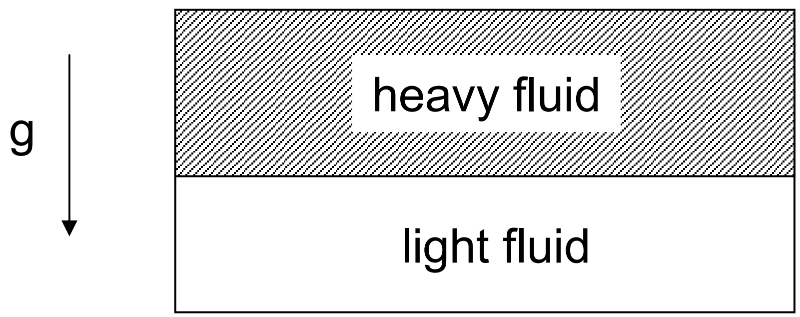
flattens in paramagnetic regions, increases paramagnetism



Magnetic phase separation drives pedestal formation

Interchange instability[1]

- present when radial force acts equally on electrons and ions
- equivalent to the Rayleigh-Taylor instability in a fluid.
- magnetization gradient acting on magnetized plasma blobs replace "gravitational field" or "curvature".



Magnetization interchange

$$\gamma = \sqrt{g\lambda_\perp} = \sqrt{-\frac{1}{m_V}\frac{\tilde{p}}{2B^2}\frac{\partial B_\zeta^2}{\partial r}\lambda_\rho}$$

Magnetization interchange growth faster for
high magnetisation, blob amplitude & radius, low field & mass

[1]M.N. Rosenbluth and C.L. Longmire, Annals of Physics, Volume 1, Issue 2, May 1957,120

Suydam criterion for interchange instability

B. R. Suydam, Proc. 2nd UN Conf. on Peaceful Uses of Atomic Energy, Geneva, 1958.

$$\beta'\left(\frac{Rq}{r_s}\right)^2\left[\frac{B^2\kappa_r}{\mu_0}\right] > \frac{q'^2}{4q^2}$$

magnetic shear opposes interchange of tubes driven by cylindrical curvature and $\nabla\beta$

Generalization:
add magnetization force to cylindrical curvature

$$\beta'\left(\frac{Rq}{r_s}\right)^2\left[\frac{B^2\kappa_r}{\mu_0} + \tilde{M}_z\frac{dB_{0z}}{dr}\right] > \frac{q'^2}{4q^2}$$

In magnetically mixed states $\tilde{M}_z\frac{dB_{0z}}{dr} < 0$

magnetisation force adds to curvature, instability,

until the magnetic shear q' or the variation of dB_z/dr changes.

Evolution towards magnetic phase transition

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As heating is applied, low pressure paramagnetic plasmas have degraded confinement, driven by low ∇p

When sufficient heating is applied, ∇p grows until zero magnetization is obtained somewhere inside the plasma: $\vec{j}_\theta = 0$

$$\nabla p_0 = \vec{j}_\zeta \times \vec{B}_\theta + \vec{j}_\theta \times \vec{B}_\zeta = 0$$

Estimate critical pressure gradient as

$$\frac{dp_0}{dr} = j_\zeta B_\theta = E_{loop} \eta_{Spitzer} B_\theta$$

Need database of typical of ∇p, loop voltage, resistivity and B_θ

to test predictions

or measurements of j_θ

Explaining T_e threshold for L-H transition via $\eta_{Spitzer}$?

and associated pressure gradient threshold

Summary and comments

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We presented a first-principles based model of plasma magnetization and magnetic phase transition as the basis for triggering confinement transitions

The magnetic state of the plasma determines convective motion of high and low pressure blobs.

Paramagnetic plasma regions attract cold blobs, become more paramagnetic.
Diamagnetic plasma regions attract hot blobs, becoming more diamagnetic.
A pedestal structure builds up in the magnetic boundary.

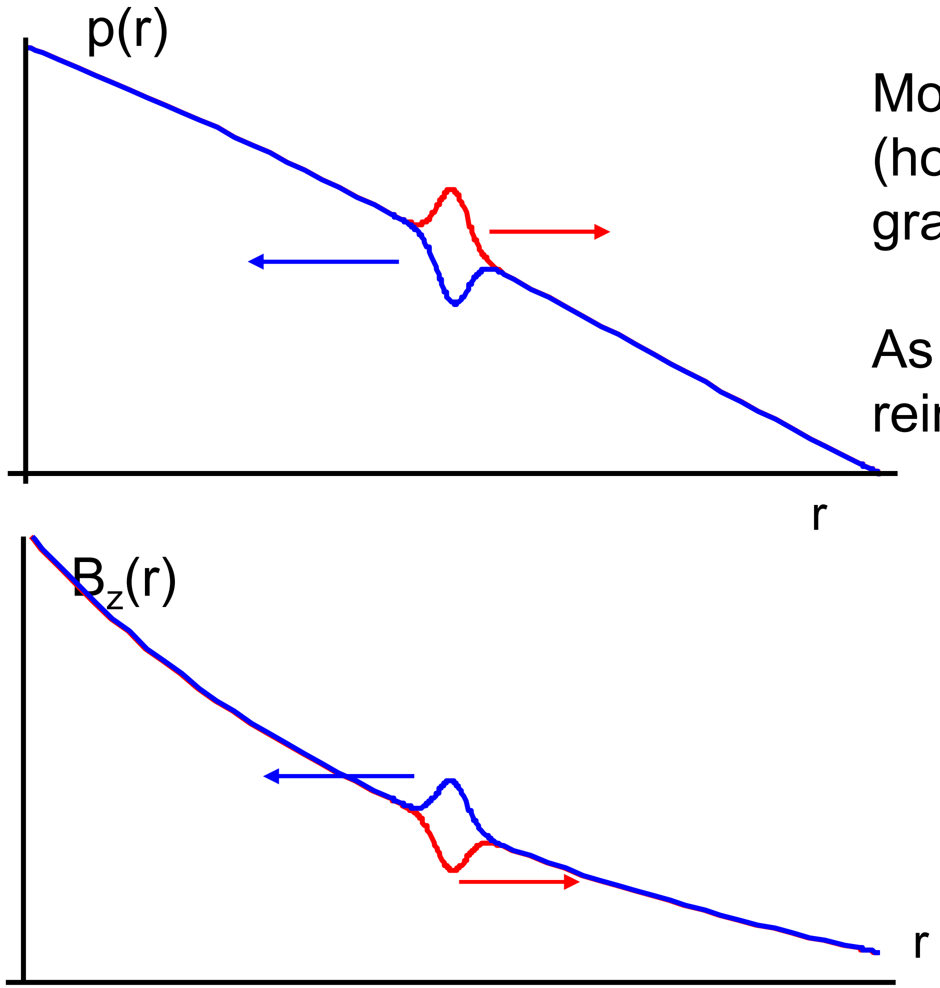
Magnetic boundary defines critical magnetization: $\mathbf{j}_\theta = 0 \Leftrightarrow \nabla \mathrm{p} = \mathbf{j}_\zeta \times \mathbf{B}_\theta$

Magnetization force drives the interchange mechanism in closed field line region, similar to interchange in SOL.

Observations and applicability conditions for model:

- in L-mode the plasma is ballooning stable, we are not studying ballooning by another name. The mechanism works in a cylinder.
- High collisionality prevents blob particles from sampling high and low field side, which is necessary so the average $\nabla \overline{B}_{0z}$ (not local) controls their behaviour. Therefore low ν^* and $\lambda_{||}>qR$ are necessary/helpful.
- Background flute-like pressure fluctuations are necessary to seed the blobs. Can be seeded by conventional interchange or other instabilities
- For j_ζ and j_θ to be non-zero at the plasma edge (so $\nabla p \neq 0$) the plasma edge temperature needs to be high enough (resistivity low)
- The pressure must be high enough to allow both positive and negative $\tilde{p}$

L mode: paramagnetic plasma



Motion of positive pressure bump (hot field-aligned blob) depends on grad(B_z).

As blobs move, magnetisation state is reinforced

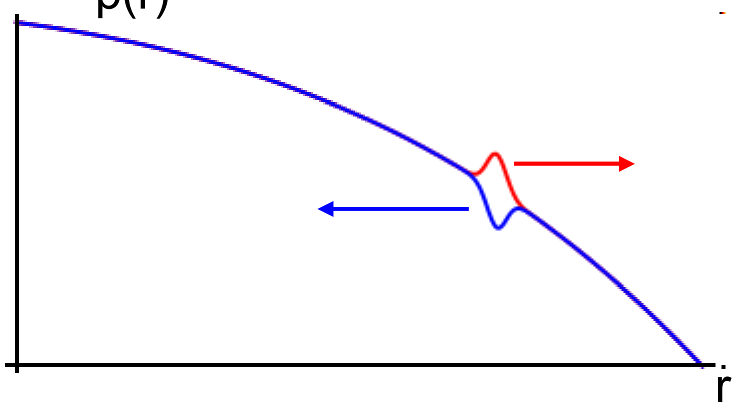
$$mn_V \frac{d\vec{v}_r}{dt} \simeq \tilde{M}_\zeta \nabla \bar{B}_{\zeta 0}$$

L mode, paramagnetic region:

Cold blobs move inward,
reduce ∇p
increase B in higher B region

Hot blobs move outward,
reduce ∇p
decrease B in lower B region.

L mode: paramagnetic plasma



Motion of positive pressure bump
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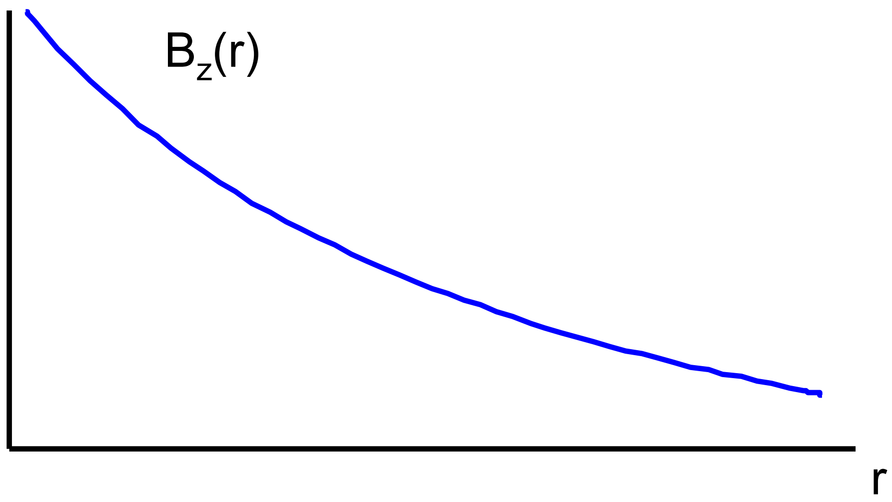
As blobs move, blob amplitude increases



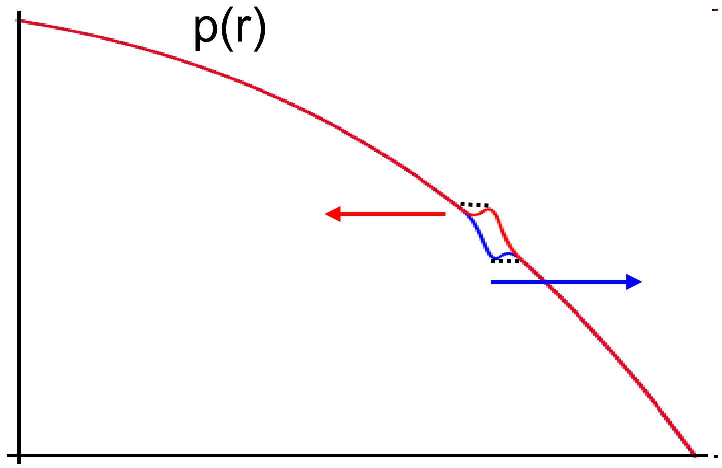
$$mn_V \frac{d\vec{v}_r}{dt} \simeq \tilde{M}_\zeta \nabla \bar{B}_{\zeta 0}$$

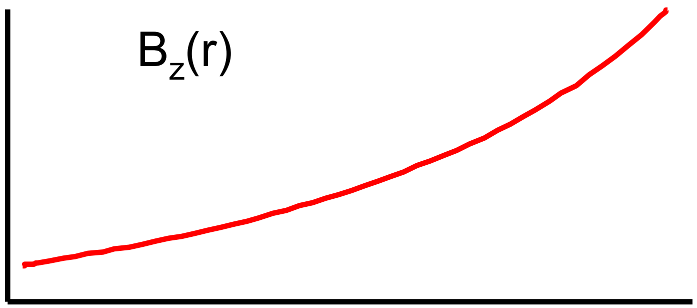
L mode, paramagnetic region:

Cold blobs move inward, up the pressure gradient, reducing it.
Hot blobs outward, reducing p gradient.

⇨ **Net outward energy convection**

H mode: diamagnetic plasma





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$$mn_V \frac{d\vec{v}_r}{dt} \simeq \tilde{M}_\zeta \nabla \bar{B}_{\zeta 0}$$

H-mode:

diamagnetic hot blobs are driven inward,
paramagnetic cold blobs outward,

until

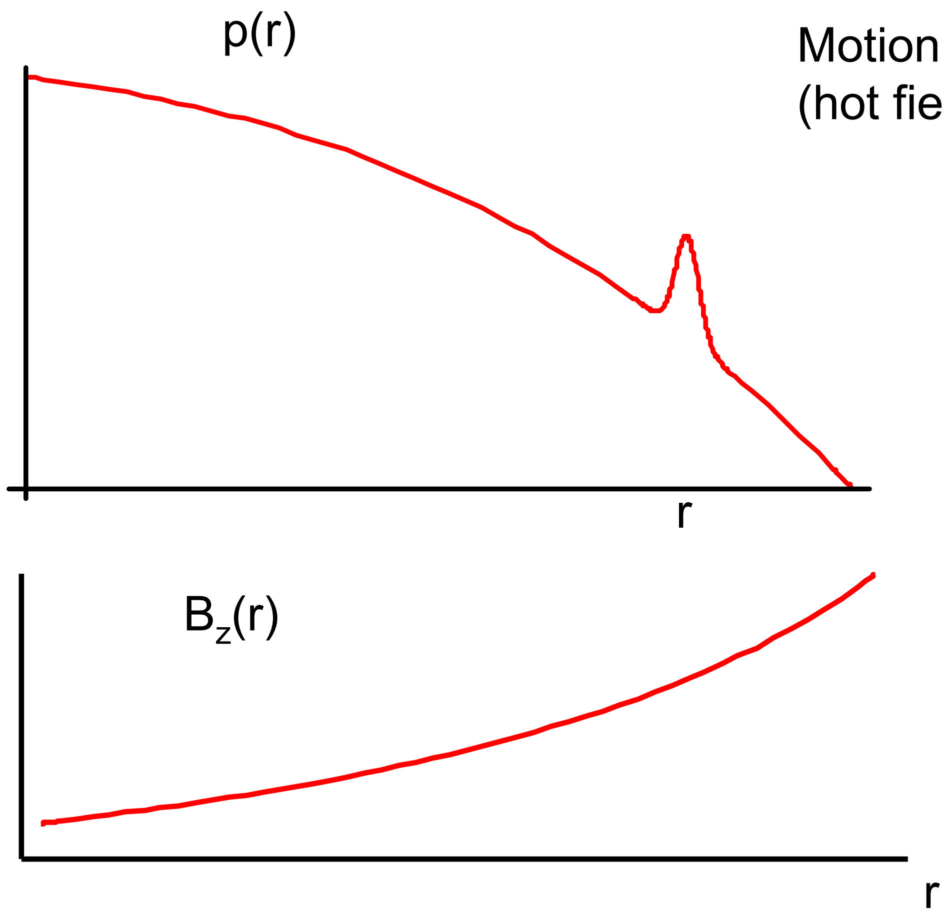
blob pressure matches background pressure,

H mode, diamagnetic region:

Cold blobs move outward, down the pressure gradient
Hot blobs move inward

⇨ **Net inward energy convection**

H mode: diamagnetic plasma



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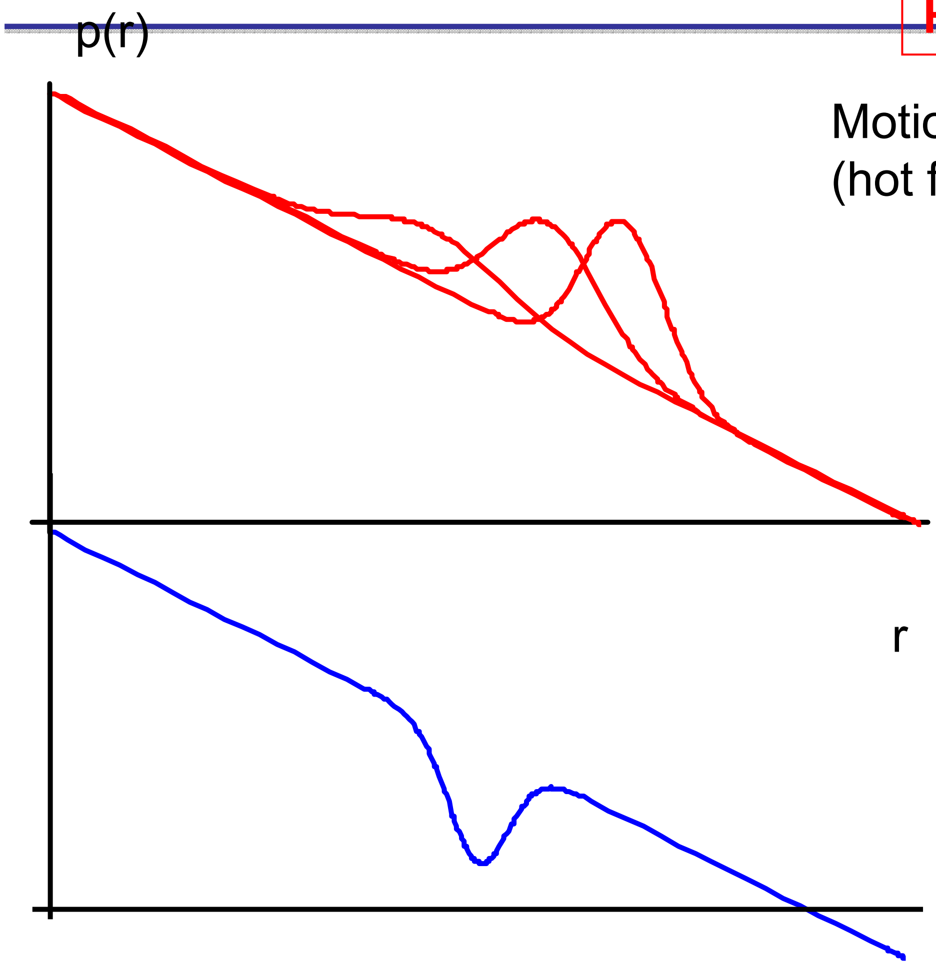

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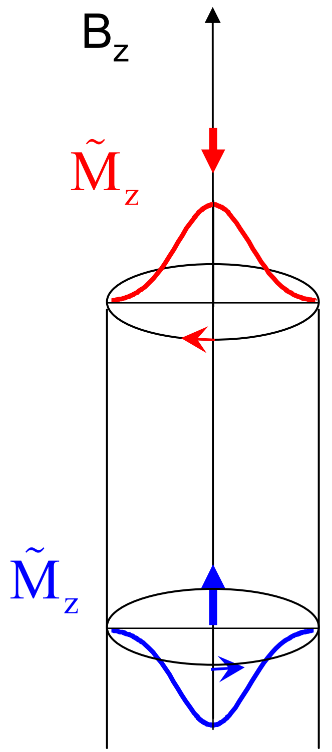

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Diamagnetic/paramagnetic current:
if inside the tube there is a pressure hill/hole,
the associated perpendicular current
reduces/increases B_z: magnetization is
negative, diamagnetism
positive, paramagnetism

Magnetization of the blob:

$$\nabla\times\vec{M}_z = \mu_0\frac{\vec{b}\times\nabla\tilde{p}}{B} = -\frac{dM_z}{dr}\hat{r}$$

$$M_z = -\frac{1}{\lambda_\parallel}\int \mu_0\lambda_\parallel\frac{\vec{b}\times\nabla\tilde{p}}{B}dr = -\mu_0\frac{\Delta\tilde{p}}{\overline{B}} \begin{cases} < 0,\ \text{dia} \\ > 0,\ \text{para} \end{cases}$$

Notice that everything here is “stable”:

The time evolution of the fluctuations is such as to reduce their amplitude (in their own frame of reference)

Nevertheless, energy flows are organised by plasma magnetisation and phase separation