Magnetic phase transition and confinement regimes

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Inspiration and motivation

- Since the discovery of the H-mode, many models of the L-H transition have been proposed
- The now conventional model is based on electrostatic turbulent vortices shredded by rotation shear. It is highly developed, can include sophisticated predator-prey model, it is now beginning to move towards electromagnetic consequences of electrostatic fluctuations…
- It is a very attractive model to many, but then I saw this movie of an L-H transition in MAST

http://www.ccfe.ac.uk/videos.aspx?currVideo=24&currCateg=0

(L-mode 10-18 s, H-mode later)

and I started thinking about phase transitions.

One of the better studied phase transitions in physics
is the magnetic phase transition.

So that got me thinking some more…

Usual set-ups

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Microturbulence	MHD
$\gamma > 0$	$\gamma > 0$
electrostatic	electro-magnetic
drift velocity	$V_{thermal}$ or $V_{Alfvén}$
Sophisticated, non-linear models	Linear models Energy principle
$\lambda \sim (1\text{-}10)\ \rho_s$	$\lambda \sim p/p'$ or $\lambda \sim a$

Usual set-ups

Microturbulence

$\gamma > 0$

electrostatic

drift velocity

Sophisticated,

non-linear models

$\lambda \sim$ (1**-10) ρ_s**

meso-scale

electromagnetic

"magnetic test-particle"

subcritical

MHD

$\gamma > 0$

electro**-magnetic**

$V_{thermal}$ or $V_{Alfvén}$

Linear models

Energy principle

$\lambda \sim p/p'$ or $\lambda \sim a$

Index

- Plasma overall magnetisation (cylindrical tokamak approximation)
- Magnetisation of “tubes” (field aligned pressure perturbations)
- Motion of magnetised tubes in magnetised plasma (B_z gradient)
- Effect on profiles
- Connections to interchange stability theory
- Experimentally testable criterion
- Data!

References:

E. R. Solano, Plasma Phys. Control. Fusion 46 L7 (2004)

E. R. Solano & R. D. Hazeltine Nucl. Fusion 52 114017 (2012)

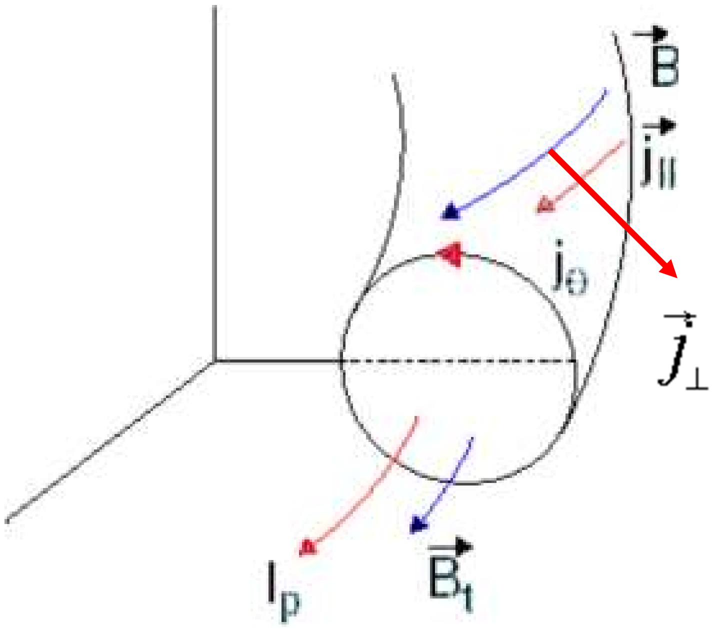
Plasma force balance:

$$\nabla p = \mathbf{j}\times\mathbf{B} = \mathbf{j}_\zeta \times \mathbf{B}_\theta + \mathbf{j}_\theta \times \mathbf{B}_\zeta$$

$$p' \equiv \frac{dp}{d\Psi}$$

$$j_\zeta = -\left(R\,p' + FF'/\left(\mu_0 R\right)\right)$$

$$\mu_0 j_\theta = -F' B_\theta \quad , \quad F(\Psi) = RB_\zeta$$



In cylindrical approximation :

$$\frac{d}{dr}\left(p + \frac{B_z^2 + B_\theta^2}{2\mu_0}\right) = -\frac{B_\theta^2}{r\mu_0}$$

$$j_z = -\left(R_0\,p' + FF'/\left(\mu_0 R_0\right)\right)$$

$$\mu_0 j_\theta = -\frac{dB_z}{dr}$$

Plasma magnetization of a "cylindrical" tokamak

Integrating cylindrical force balance:

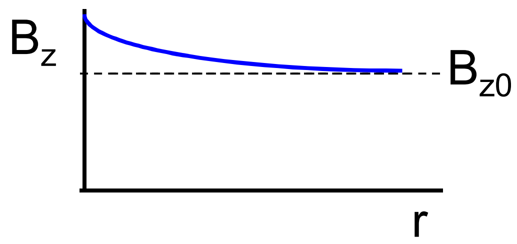
$$\beta_\theta \equiv \frac{\int_0^a p dS}{B_{\theta a}^2 / 2\mu_0} = \frac{B_{za}^2 - \langle B_z^2 \rangle}{B_{\theta a}^2} \simeq 1 + \frac{2B_{za}\left(B_{za} - \langle B_z \rangle\right)}{B_{\theta a}^2}$$

$$\frac{dB_z}{dr} = -\mu_0 j_\theta$$

(β_θ -1) is related to normalised average plasma magnetisation

β_θ <1 — B_z increased by j_θ
paramagnetism,
low pressure

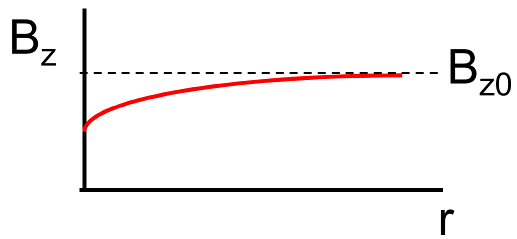
$$\nabla p < \vec{j}_\zeta \times \vec{B}_\theta$$

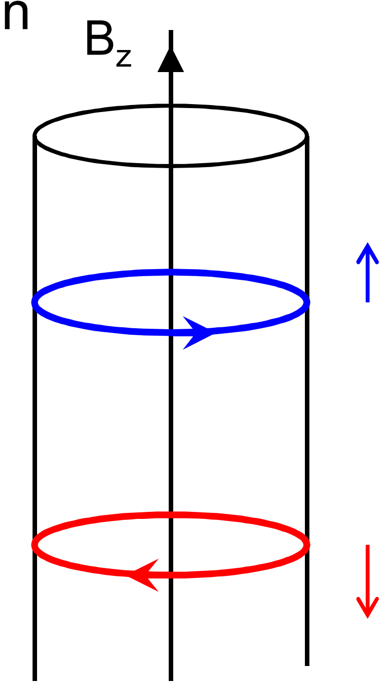


β_θ> 1 — B_z reduced by j_θ
diamagnetism,

$$\nabla p > \vec{j}_\zeta \times \vec{B}_\theta$$

high pressure





MAGNETS

The tokamak plasma is a magnet.

$$\langle B_z \rangle - B_z^{vac} \cong \mu_0 \left(B_{\theta a}^2 / 2\mu_0 - \int_0^a p dS \right) \Big/ B_z^{vac}$$

the difference between poloidal magnetic and kinetic pressure determines if it is a para-magnet or a dia-magnet

Paramagnets

increase the background magnetic field
move towards high field regions

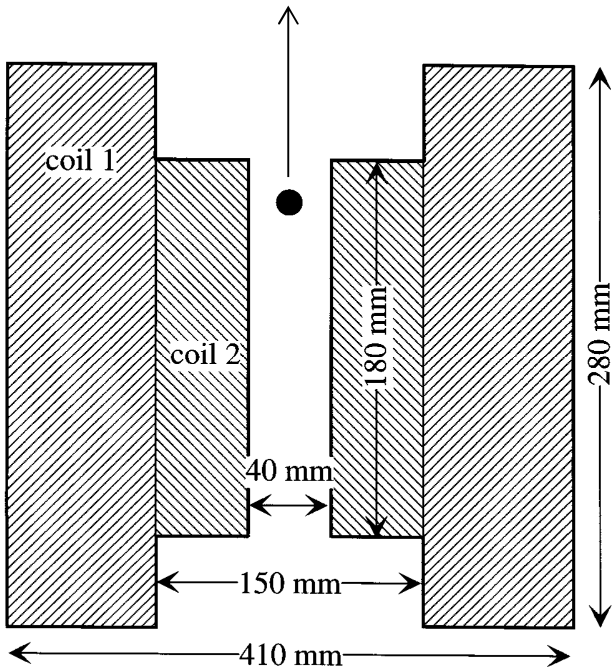
Diamagnets

decrease the background magnetic field
move towards low field regions

so far I have just reviewed well known facts

Diamagnetic levitation

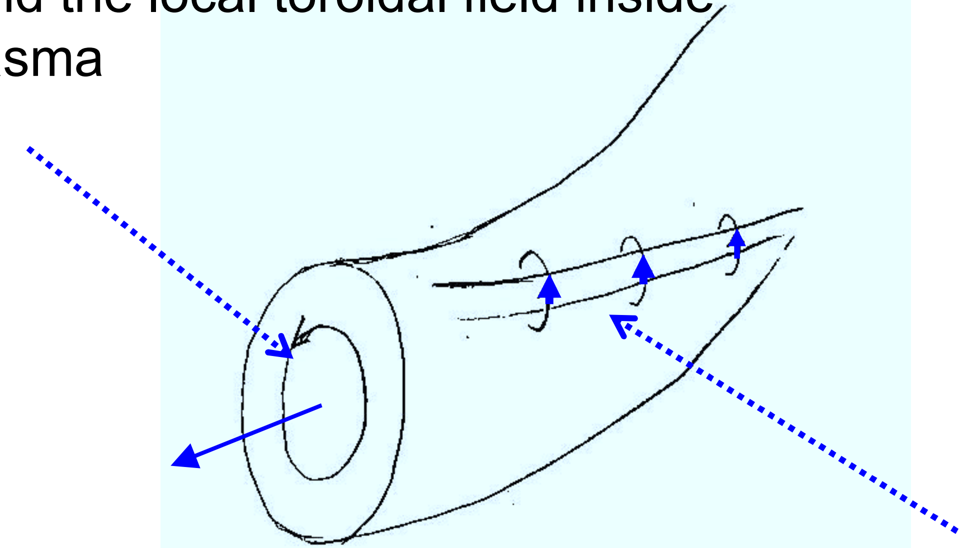
A frog (diamagnetic) dropped in a strong magnetic field levitates because it tries to get away from the high field. It moves towards the lower field, arranged to be upwards.



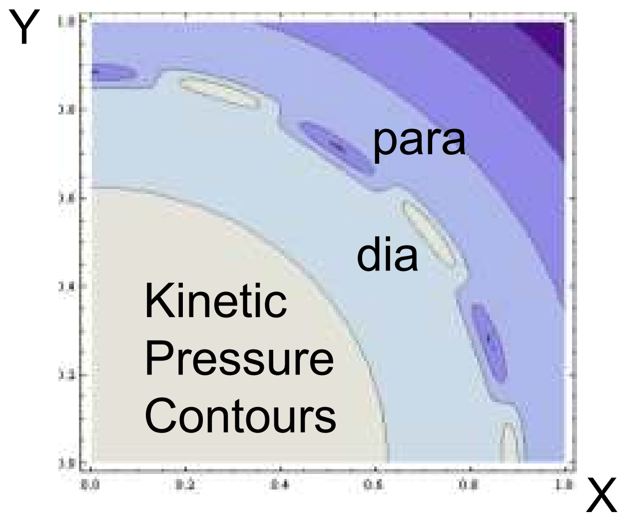
M V Berry and A K Geimz
Eur. J. Phys. 18 (1997) 307–313.

Magnetised plasma element

The poloidal current density around a tokamak plasma is responsible for plasma magnetisation: the difference between the externally applied toroidal field and the local toroidal field inside the plasma



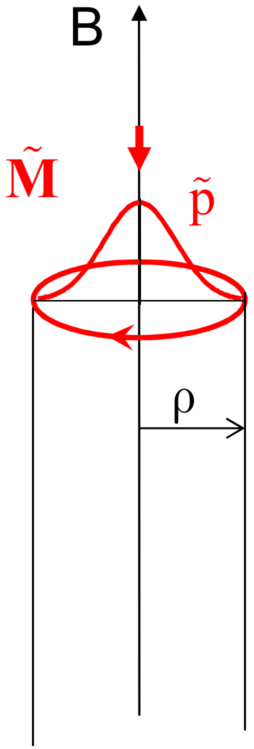
Not conventional current "filaments", with parallel current density



Next consider a field-aligned plasma element with a pressure perturbation relative to the background plasma pressure.

What is its magnetisation?

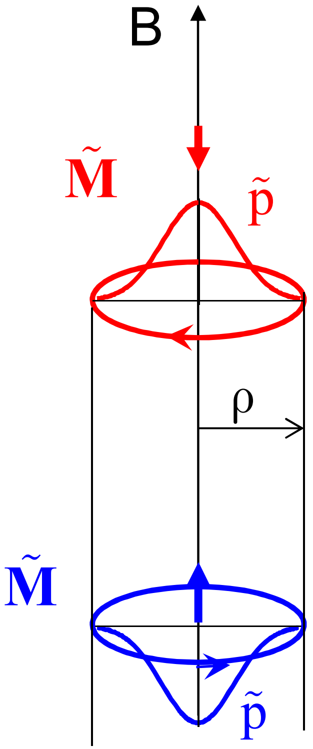
Magnetism in cylindrical tube with pressure hill/hole



$$\mathbf{F}_\rho = mn\frac{d\mathbf{v}_\rho}{dt} = -\nabla_\rho \tilde{p} + \left(\tilde{\mathbf{j}} \times \mathbf{B}\right)_\rho \qquad \tilde{\mathbf{j}}_\perp = \frac{\mathbf{b} \times \nabla \tilde{p}}{B}$$

Diamagnetic current:
if inside the tube there is a pressure hill (more pressure than in the background plasma), the associated perpendicular current reduces B_z: diamagnetism

Magnetism in cylindrical tube with pressure hill/hole



$$\mathbf{F}_\rho = mn\frac{d\mathbf{v}_\rho}{dt} = -\nabla_\rho \tilde{p} + \left(\tilde{\mathbf{j}} \times \mathbf{B}\right)_\rho \qquad \tilde{\mathbf{j}}_\perp = \frac{\mathbf{b} \times \nabla \tilde{p}}{B}$$

Diamagnetic current:
if inside the tube there is a pressure hill, the associated perpendicular current reduces B_z: diamagnetism

Paramagnetic current:
if inside the tube there is a pressure hole (less pressure than in the background plasma), the associated perpendicular current increases B_z: paramagnetism

Magnetism in cylindrical tube with pressure hill/hole

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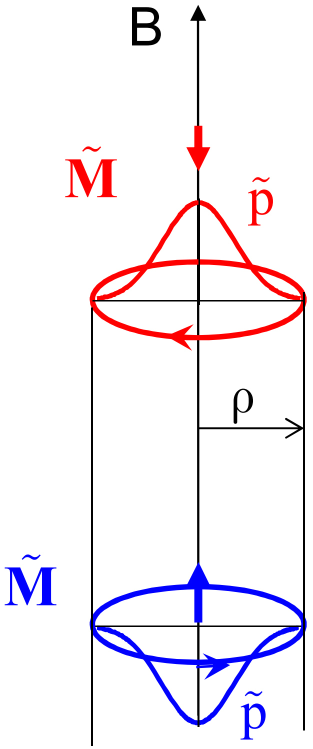


$$\mathbf{F}_\rho = mn\frac{d\mathbf{v}_\rho}{dt} = -\nabla_\rho \tilde{p} + \left(\tilde{\mathbf{j}} \times \mathbf{B}\right)_\rho \qquad \tilde{\mathbf{j}}_\perp = \frac{\mathbf{b} \times \nabla \tilde{p}}{B}$$

Diamagnetic current:
if inside the tube there is a pressure hill, the associated perpendicular current reduces B_z: diamagnetism

Paramagnetic current:
if inside the tube there is a pressure hole, the associated perpendicular current increases B_z: paramagnetism

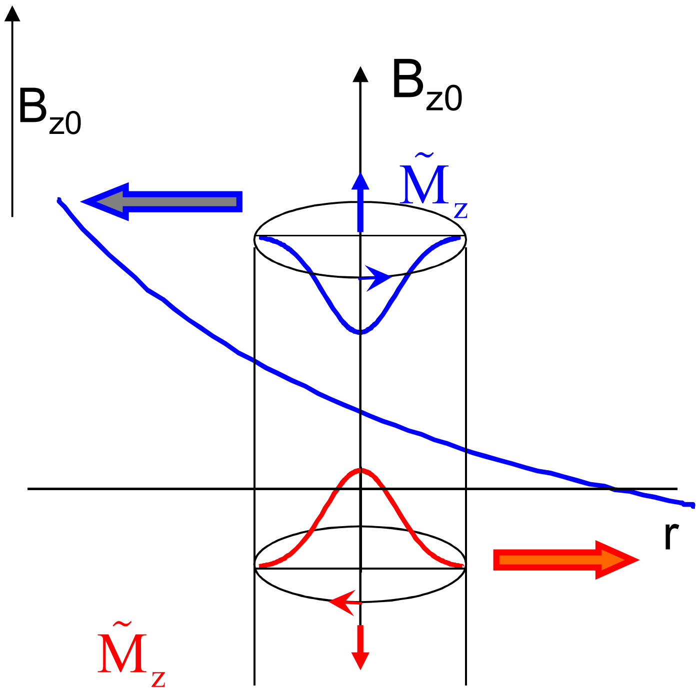
Magnetization of the blob:

$$\nabla \times \tilde{\mathbf{M}} = \mu_0 \frac{b \times \nabla \tilde{p}}{B} = -\frac{d\tilde{M}}{dr}\hat{\mathbf{r}}$$

$$\tilde{\mathbf{M}} = \frac{1}{\lambda_\parallel}\int_0^\rho \frac{\mathbf{b}}{B}\frac{\partial \tilde{p}(\rho')}{\partial \rho'}\lambda_\parallel \, d\rho' \approx -\frac{\tilde{p}}{\bar{B}}\mathbf{b} \quad \begin{cases} < 0, \text{ dia} \\ > 0, \text{ para} \end{cases}$$

Movement of magnetized object in field gradient

(see Jackson)



$$m_V \frac{d\mathbf{v}}{dt}\bigg|_V = \int \left(\nabla(\tilde{\mathbf{M}}.\mathbf{B})\right) dV \simeq$$

$$\vec{B} = \vec{B}_0 + \vec{r}.\nabla\vec{B}_0 + \ldots$$

$$\simeq \underbrace{\left(\int \left(\mathbf{r} \times \tilde{\mathbf{j}}\right) dV\right)}_{\text{blob magnetization}} \int \nabla B_{0z} dV$$

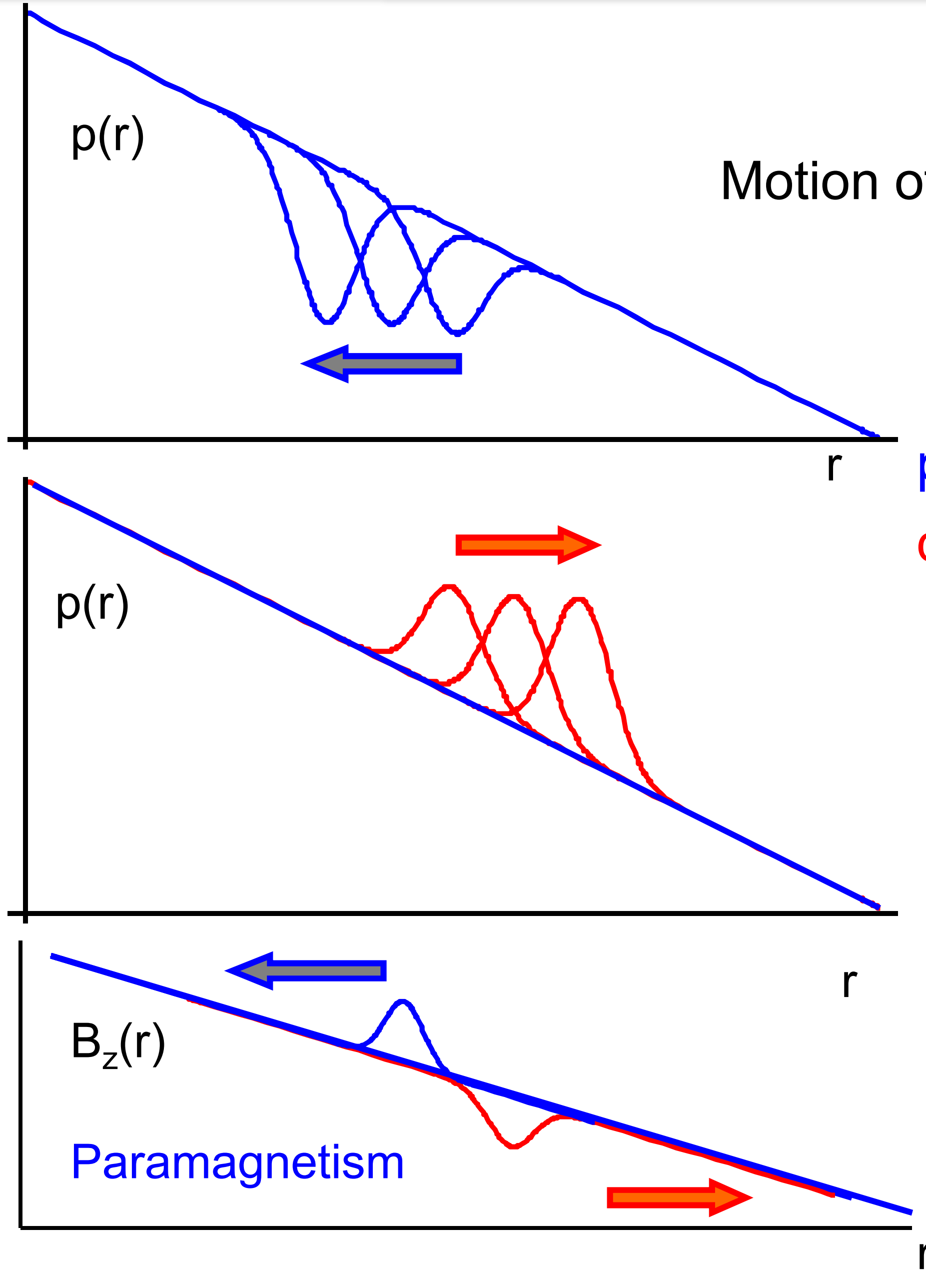
$$mn_V \frac{d\mathbf{v}_r}{dt} \simeq \tilde{M}_z \nabla \bar{B}_{z0} = -\mu_0 \tilde{M}_z \bar{j}_\theta$$

the cold tube (paramagnetic) seeks high field

the hot tube (diamagnetic) seeks low field

averaged dB_z/dr controls motion of magnetised plasma tubes:
Anti-potential leads to ***magnetic phase separation***

Paramagnetic plasma: L-mode



Motion of pressure blobs depends on dB_z/dr

$$mn_V \frac{dv_r}{dt} \simeq \tilde{M}_\zeta \frac{d\bar{B}_{\zeta 0}}{dr}$$

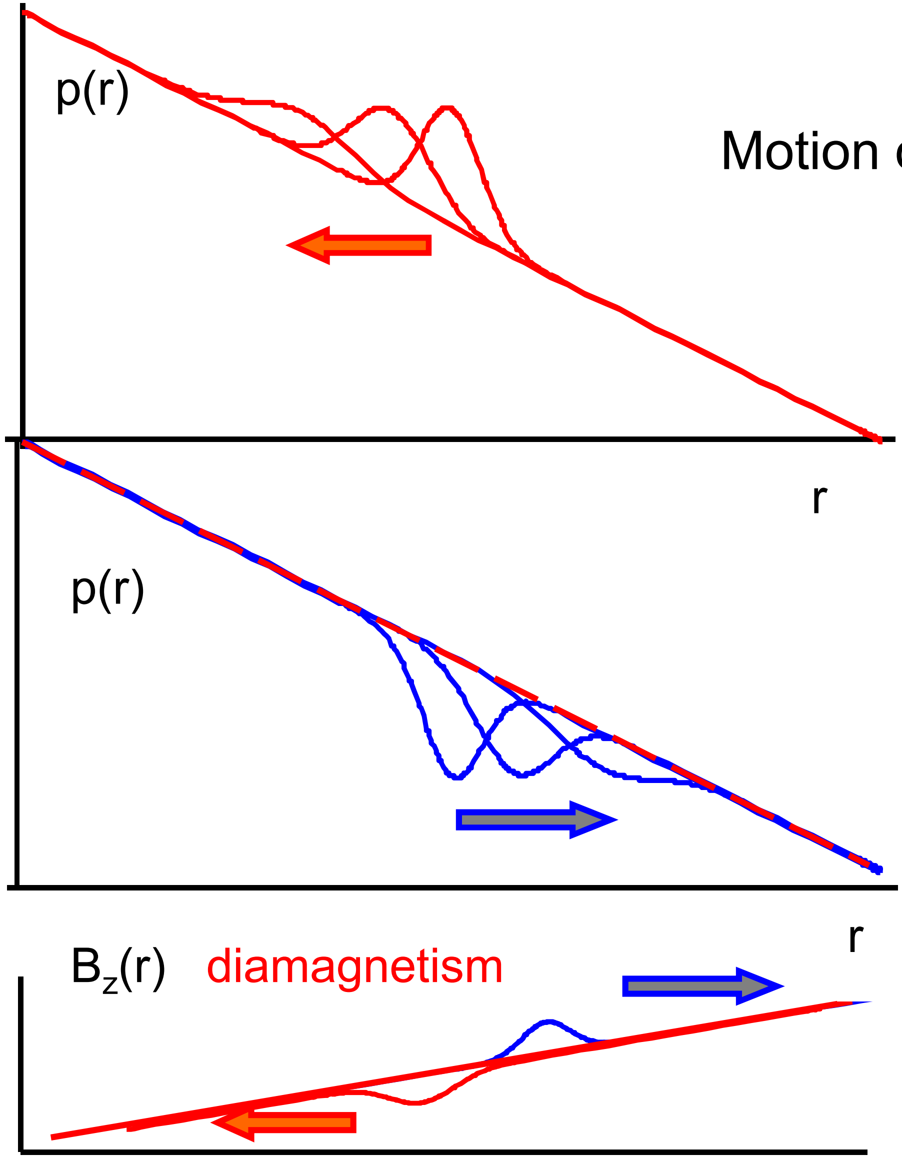
paramagnetic cold blobs move inward,
diamagnetic hot blobs move outward

outward thermal energy convection
at the expense of
inward magnetic energy convection

$\tilde{p}$ blobs "grow", "instability"

L-mode

Diamagnetic plasma: H-mode



Motion of pressure blobs depends on dB_z/dr

$$mn_V \frac{dv_r}{dt} \simeq \tilde{M}_\zeta \frac{d\bar{B}_{\zeta 0}}{dr}$$

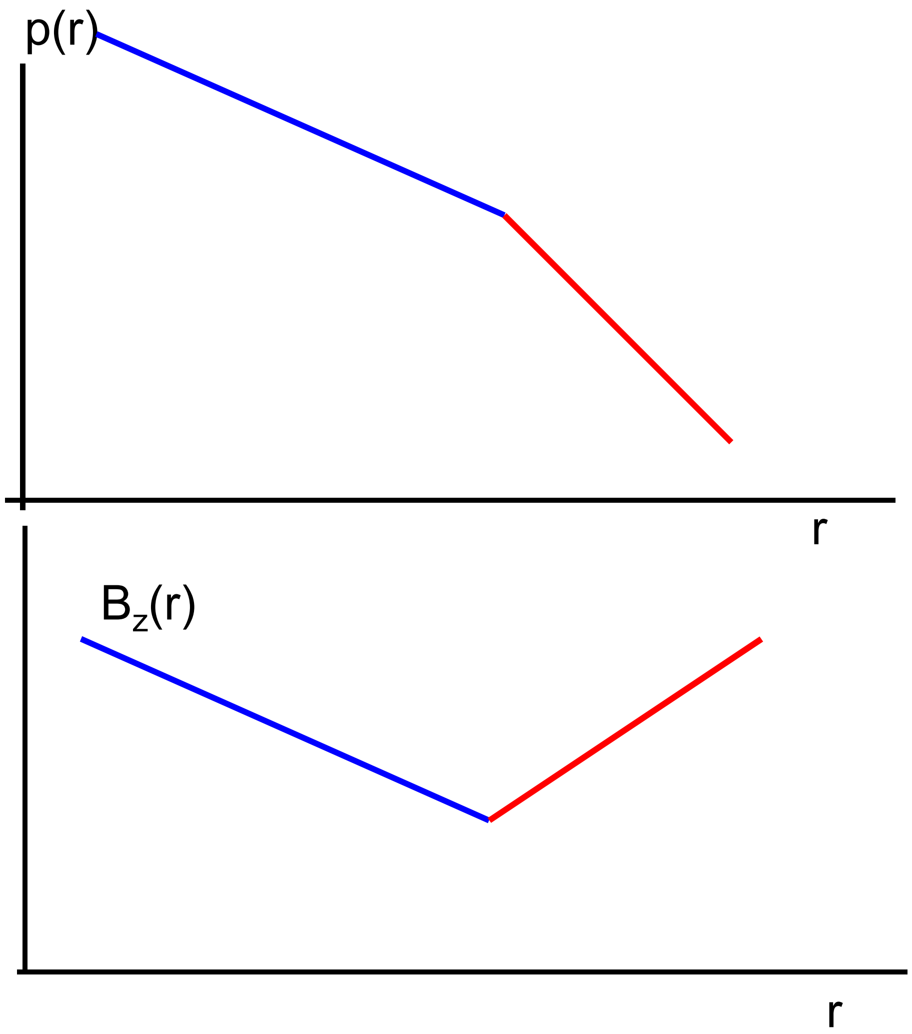
diamagnetic hot blobs move inward,
paramagnetic cold blobs move outward

inward thermal energy convection
at the expense of
outward magnetic energy convection

$\tilde{p}$ blobs "decrease", "saturation"

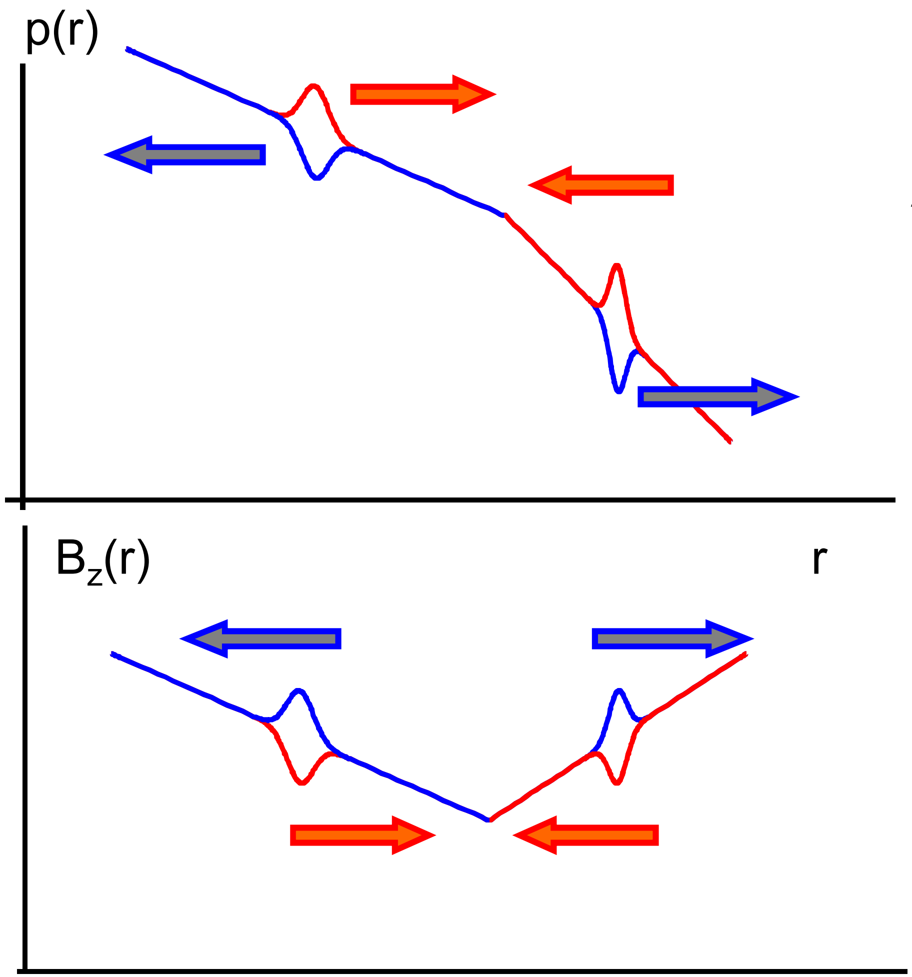
H-mode

Magnetic Boundary: phase transition



∇p increases somewhere, creating diamagnetic region at plasma edge.

Magnetic Boundary: phase transition



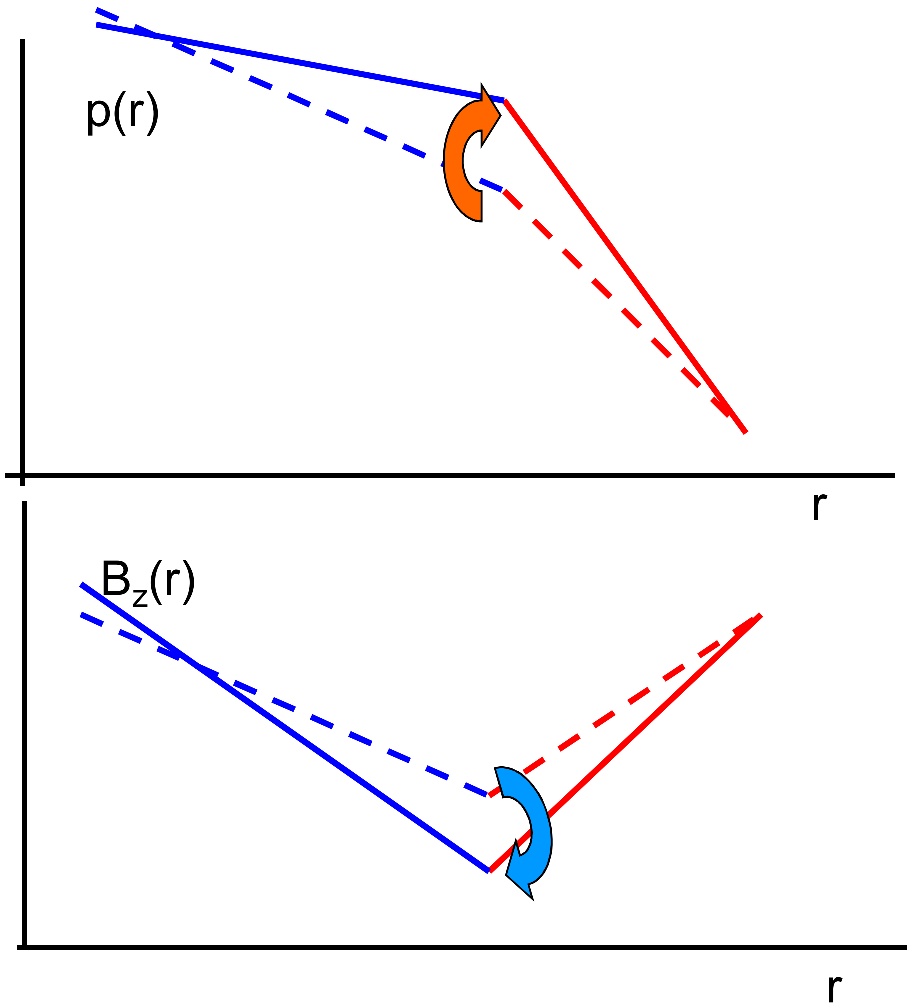
At a magnetic phase boundary blobs of the same type accumulate

diamagnetic blobs (heat) seek wells

paramagnetic blobs seek hilltops

With multiple blobs moving, p and B_z profiles evolve

Magnetic Boundary: phase transition



Pressure gradient

increases in diamagnetic region

Decreases in paramagnetic region

Magnetization,

of both signs, increases.

Phase transition is self-reinforcing.

Pressure pedestal forms, grows.

Evolution equations

Ideal 1 fluid MHD evolution

<u>magnetization force terms</u>

$$n m_i \frac{d\mathbf{v}}{dt} = \mathbf{F} \longrightarrow \bar{n}_V m_i \frac{dv_r}{dt}\bigg|_M = \tilde{M}_\zeta \frac{d}{dr} \bar{B}_{0z}$$

$$\frac{3}{2}\frac{\partial p}{\partial t} = -\nabla.\mathbf{Q} + H \longrightarrow \frac{3}{2}\frac{\partial p}{\partial t}\bigg|_M = -\frac{d}{dr}(\tilde{p}\, v_r)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) - \nabla\times\eta\left(\mathbf{j}-\mathbf{j}_{ni}\right) \longrightarrow \frac{\partial \mathbf{B}_z}{\partial t}\bigg|_M = \nabla\times\left(v_r B_z \boldsymbol{\theta}\right)$$

For now, consider what the magnetisation force does, disregard other transport mechanisms

Pedestal formation at magnetisation boundary

Assume dashed $B_z(r)$, $p(r)$ initial profiles

Ideal MHD with magnetization force

$$\bar{n}_V m_i \left.\frac{dv_r}{dt}\right|_M = \tilde{M}_\zeta \frac{d}{dr}\bar{B}_{0z}$$

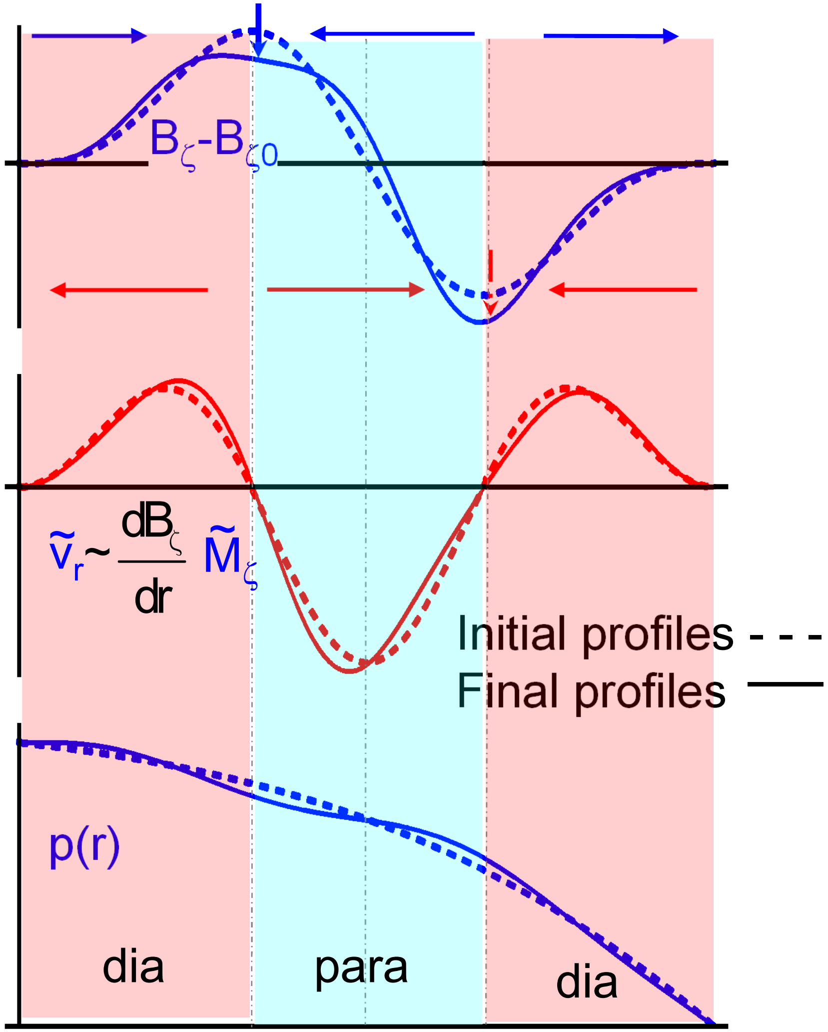
$$\frac{3}{2}\left.\frac{\partial p}{\partial t}\right|_M = -\frac{d}{dr}(\tilde{p}\, v_r)$$

$$\left.\frac{\partial B_z}{\partial t}\right|_M = \frac{d}{dr}(\tilde{v}_r \bar{B}_{0z})$$

Integrating one temporal step Δt

pressure steepens in diamagnetic regions, increases diamagnetism

flattens in paramagnetic regions, increases paramagnetism



Magnetic phase separation drives pedestal formation

Recap and applicability conditions

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- plasma tubes with an excess or defect of pressure are convected radially, depending on the plasma magnetisation.
- phase separation occurs at the flux surface in which j_q changes sign.

Under what conditions?

- *The seed pressure perturbation is strong enough to protrude above or below the background pressure profile.*
- *plasma elements must be long enough to average out the 1/R variation of the vacuum field: $\lambda_{||} > qR$. Otherwise conventional, ballooning-like transport would drive short diamagnets towards low R.*
- *Collisionality/resistivity: the temperature must be high enough for particles to sample LFS and HFS before being scattered out of the tube.*
- *Edge pressure must be high enough to allow negative perturbations as well as positive.*

Caveats

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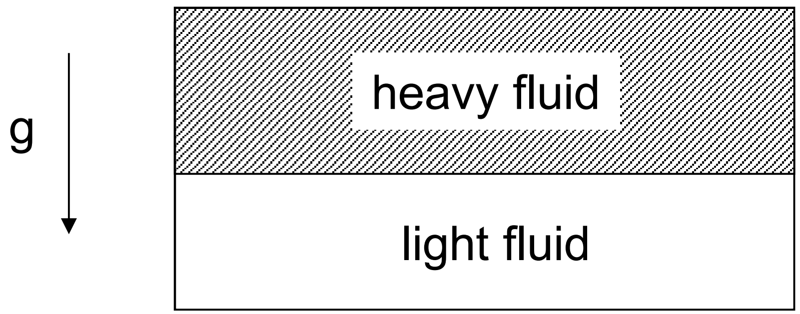
- So far we have treated the plasma tubes as “test particles” with a magnetic moment
- ignored geometrical magnetisation from $j_{||}$
- No evolution equation for tube magnetisation.
- the magnetic interaction between plasma elements and bulk plasma is quite complex, and our model very simple (too simple?)
- We hope that a more detailed calculation, up to second order on the spatial variation of $F=RB_{tor}$, can be carried out. Kind of neoclassical magnetisation, instead of classical
- Much harder to do …

Interchange instability[1]

- present when a radial force acts equally on electrons and ions
- equivalent to the Rayleigh-Taylor instability in a fluid.
- magnetization gradient acting on magnetized plasma blobs replace "gravitational field" or "curvature".

Magnetization interchange



$$\gamma = \sqrt{g/\lambda_\perp}$$

$$\gamma = \sqrt{-\frac{1}{\bar{\rho}_{m,blob}} \frac{\tilde{p}}{\bar{B}} \frac{d\bar{B}_\zeta}{dr} \frac{1}{\lambda_\rho}}$$

Magnetization interchange growth faster for
high magnetisation, strong seed, low field & mass

[1]M.N. Rosenbluth and C.L. Longmire, Annals of Physics, Volume 1, Issue 2, May 1957,120

Suydam criterion for interchange instability

B. R. Suydam, Proc. 2nd UN Conf. on Peaceful Uses of Atomic Energy, Geneva, 1958.

$$\beta'\left(\frac{Rq}{r_s}\right)^2\left[\frac{B^2\kappa_r}{\mu_0}\right] > \frac{q'^2}{4q^2}$$

magnetic shear opposes interchange of tubes driven by cylindrical curvature and $\nabla\beta$

Generalization:
add magnetization force to cylindrical curvature

$$\beta'\left(\frac{Rq}{r_s}\right)^2\left[\frac{B^2\kappa_r}{\mu_0} + \tilde{M}_z\frac{dB_{0z}}{dr}\right] > \frac{q'^2}{4q^2}$$

In magnetically mixed states $\tilde{M}_z\frac{dB_{0z}}{dr} < 0$

magnetisation force adds to curvature, instability,

until the magnetic shear q' or the variation of dB_z/dr changes.

Evolution towards magnetic phase transition

As heating is applied, low pressure paramagnetic plasmas have degraded confinement, driven by low ∇p

When sufficient heating is applied, ∇p grows until zero magnetization is obtained somewhere inside the plasma: $j_\theta = 0$

$$\nabla p = \mathbf{j}_\zeta \times \mathbf{B}_\theta + \mathbf{j}_\theta \times \mathbf{B}_\zeta = 0$$

Estimate critical pressure gradient as

$$\frac{dp}{dr} = j_\zeta B_\theta = E_{loop} \eta_{Spitzer} B_\theta$$

Need database of typical ∇p, loop voltage, resistivity and B_θ

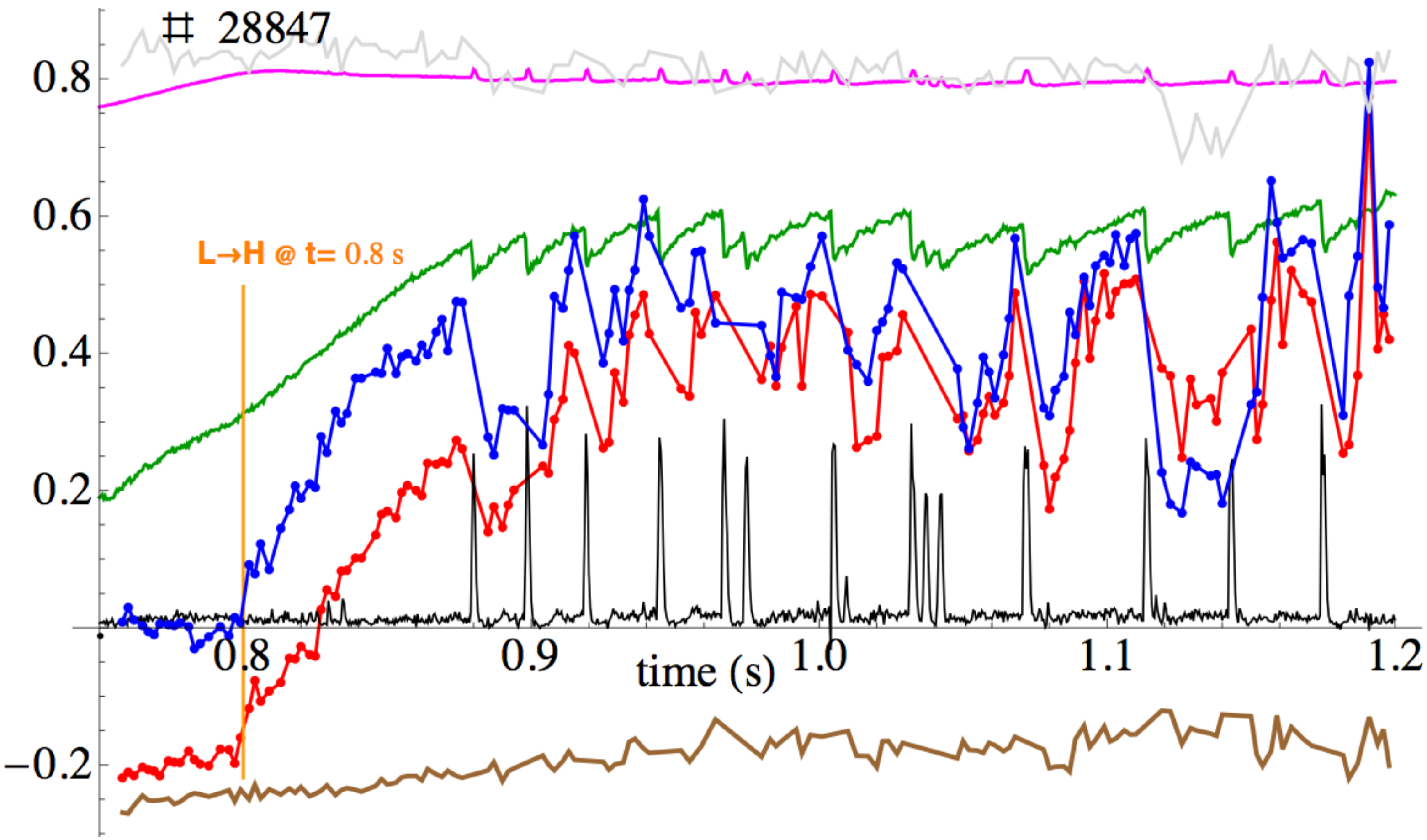
to test predictions

or measurements of j_θ

Explaining T_e threshold for L-H transition via $\eta_{Spitzer}$?

and associated pressure gradient (β_θ) threshold

Experimental evidence? AUG



(10×) j_θ zero crossing at L→H transition

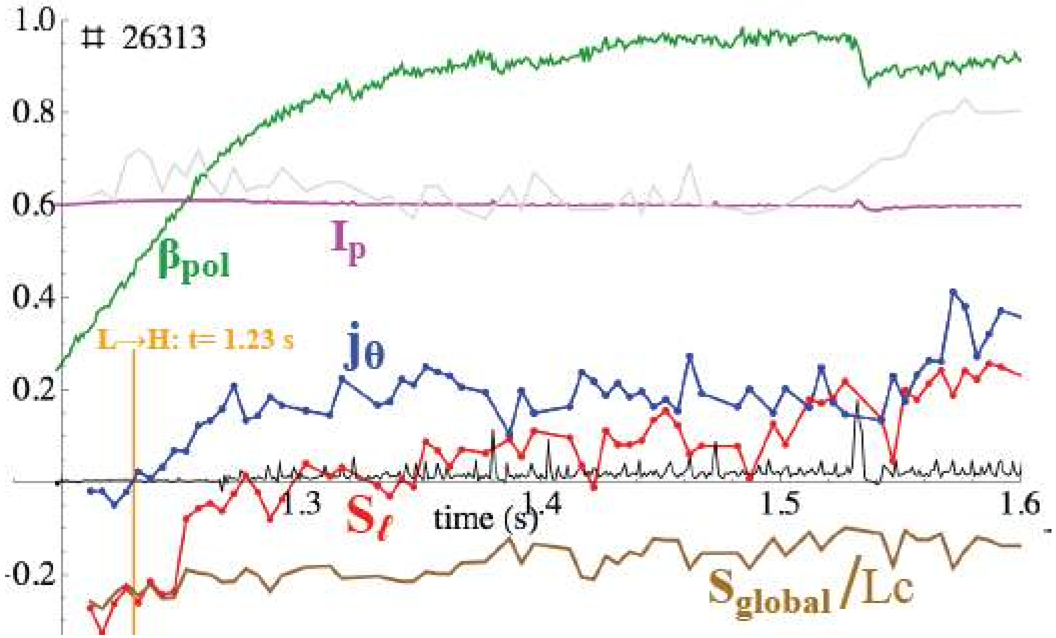
P. J. McCarthy, P4.115, 40th EPS Conference on Plasma Physics, Espoo, 2013

Summary statistics (ms) for 10 discharges

TIME DELAY	Mean	Std Dev	Min	Max
$t_{j_\theta=0} \rightarrow t_{L\text{-}H}$	3	7	-11	15

Experimental evidence? AUG II

In a slow L-H transition followed by "type III" ELMs
j_θ remains diamagnetic after transition



But at least one counter example has been found in a slow transition (still unpublished). More analysis needed, as well as more refined model.

Summary and comments

- First-principles model of plasma magnetization and magnetic phase transition as the basis for triggering confinement transitions
- The magnetic state of the plasma determines convective motion of high and low pressure tubes.
- Paramagnetic plasma regions attract cold tubes, become more paramagnetic.
- Diamagnetic plasma regions attract hot tubes, becoming more diamagnetic.
- A pedestal structure builds up in the magnetic phase boundary.
- Magnetic boundary defines critical magnetization: $\mathbf{j}_\theta = 0 \Leftrightarrow \nabla p = \mathbf{j}_\zeta \times \mathbf{B}_\theta$
- Magnetization force drives the magnetic interchange mechanism in closed field line region, similar to curvature interchange in SOL.